Lecture 9
2025/2026

# Microwave Devices and Circuits for Radiocommunications

# 2025/2026

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday **12-14**, **P2**
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 24.02.2026 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

# 2025/2026

- Laboratory – **associate professor Radu Damian**
  - Monday 14-16, II.13 / (even weeks)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 24.02.2026)
    - personal homework

General theory

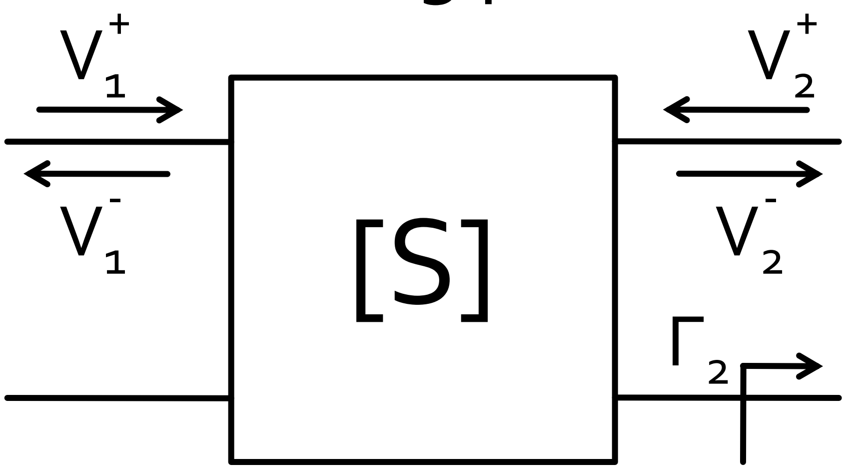
# Microwave Network Analysis

# Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

# Scattering matrix – S

- Scattering parameters




$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} \qquad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

# Power waves for N ports

$$[b]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}\cdot[a]$$

- The scattering matrix for power waves, $[S_p]$

$$[b]=[S_p]\cdot[a]$$

$$[S_p]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}$$

- But: $[S]=\left([Z]-[Z_0]\right)\cdot\left([Z]+[Z_0]\right)^{-1}$

- Typically

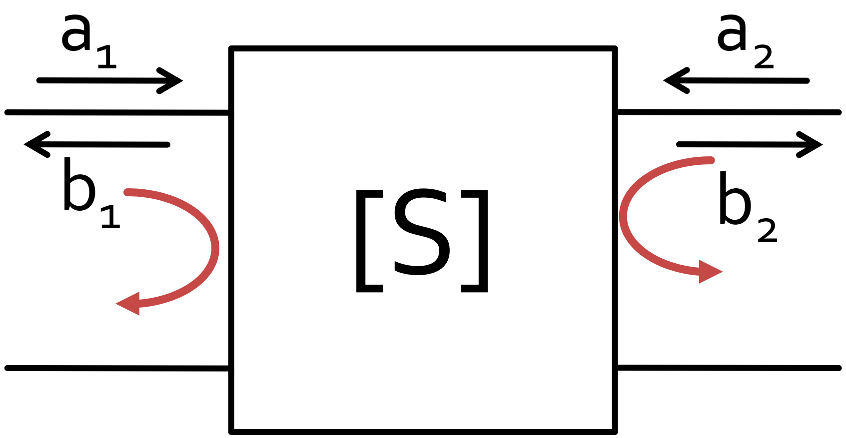
$$Z_{0i}=Z_{Ri}=R_0,\forall i$$

$$R_0=50\Omega$$

$$[S_p]\equiv[S]$$

- **they coincide!!!**

# Scattering matrix – S

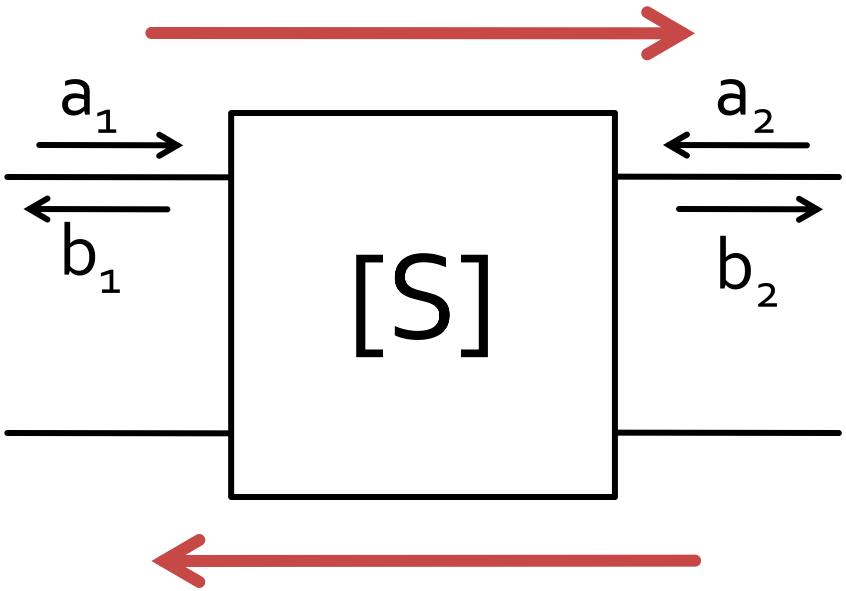



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

- $S_{11}$ and $S_{22}$ are **reflection coefficients** at ports 1 and 2 when the other port is **matched**

# Scattering matrix – S

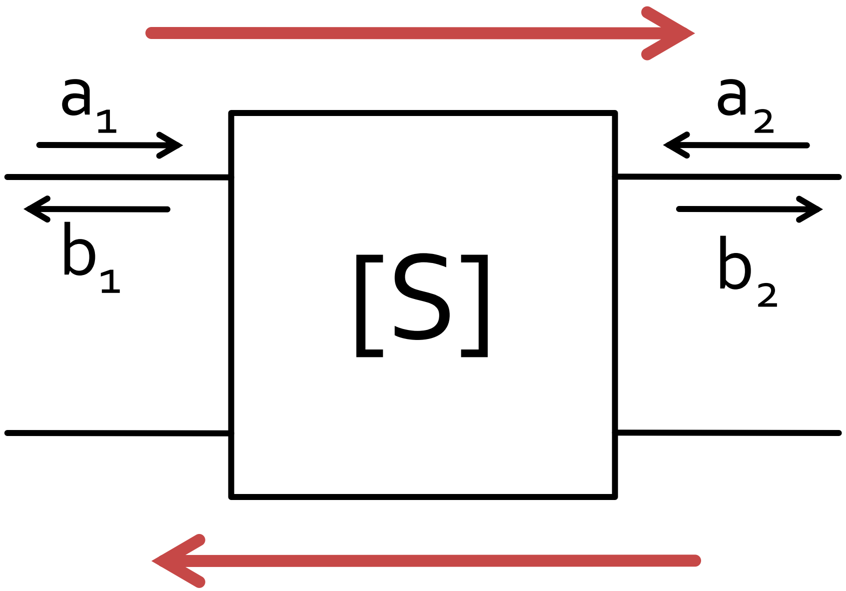



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \qquad S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

- $S_{21}$ si $S_{12}$ are signal amplitude **gain** when the other port is **matched**

# Scattering matrix – S




$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- $S_{ij}$
  - network effect (gain) over signal power **including** phase information

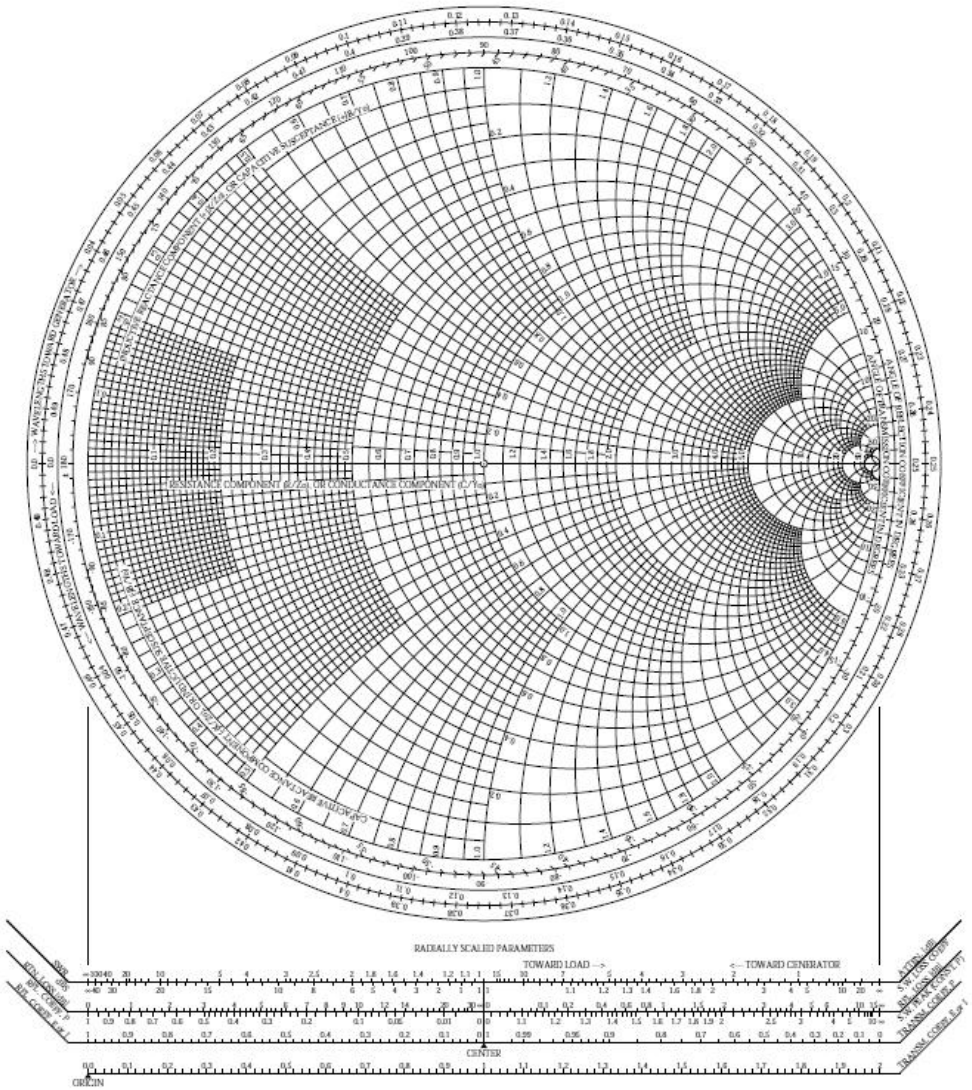
Impedance Matching

# The Smith Chart

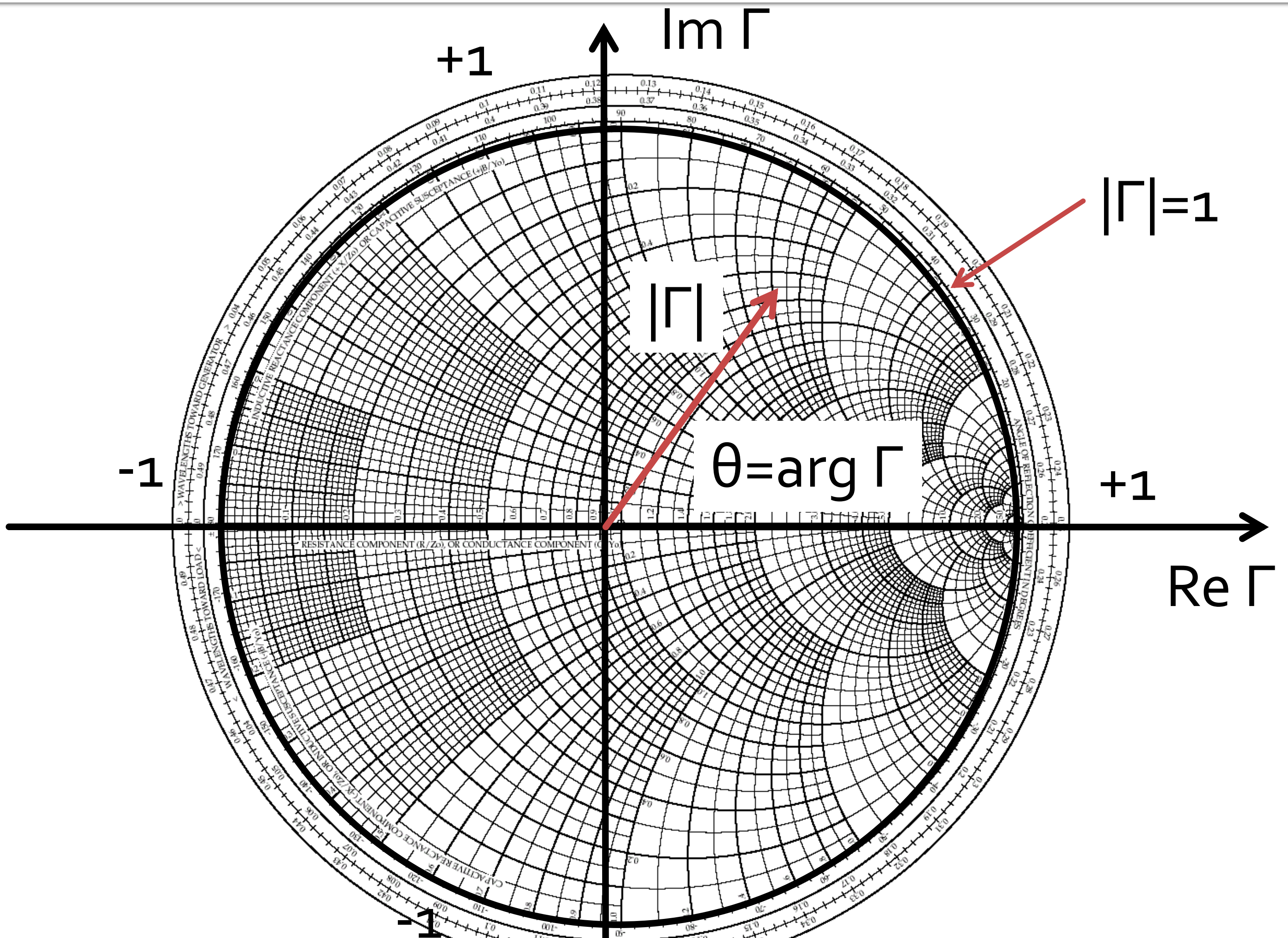
# Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

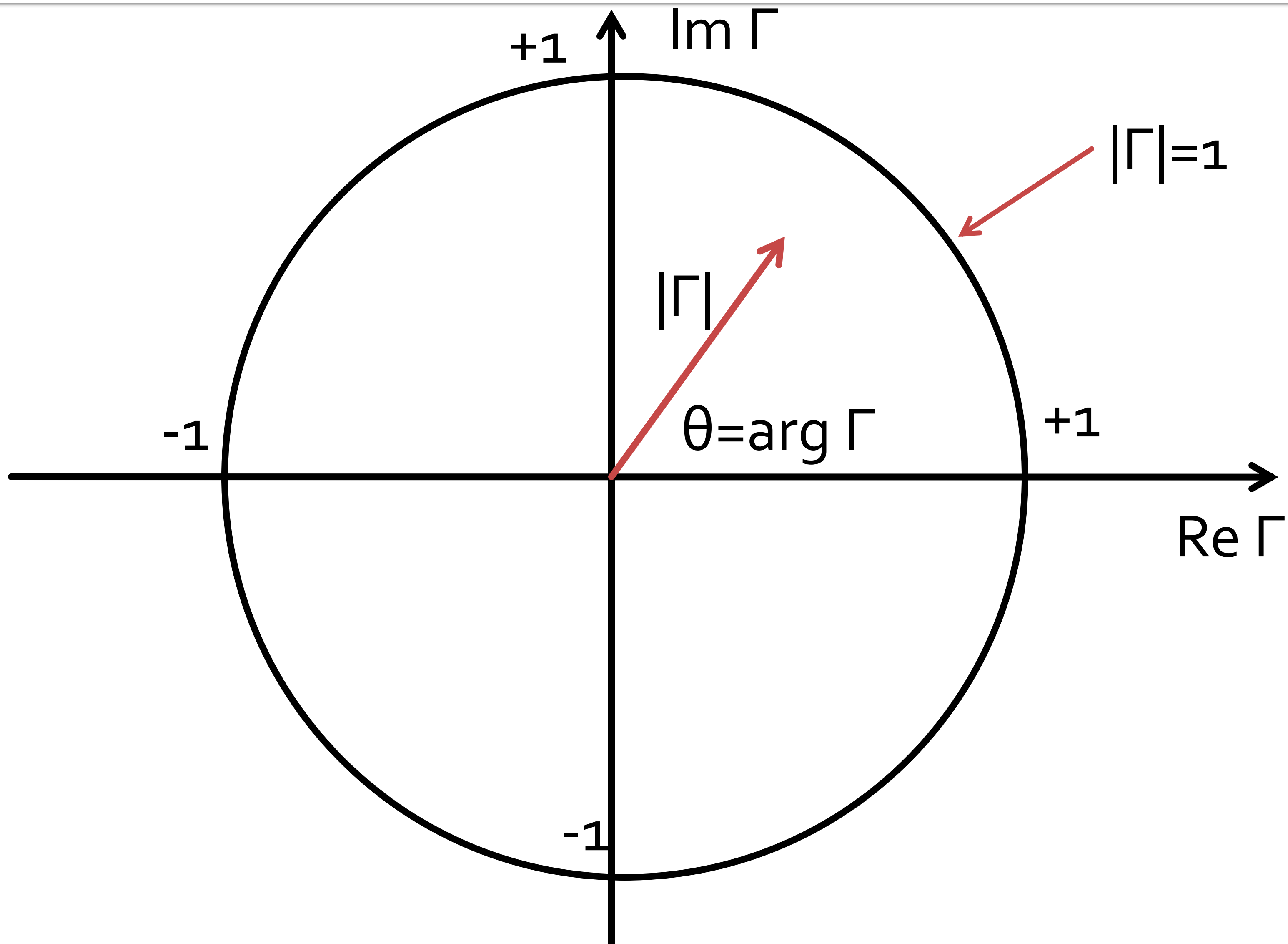
# The Smith Chart



# The Smith Chart

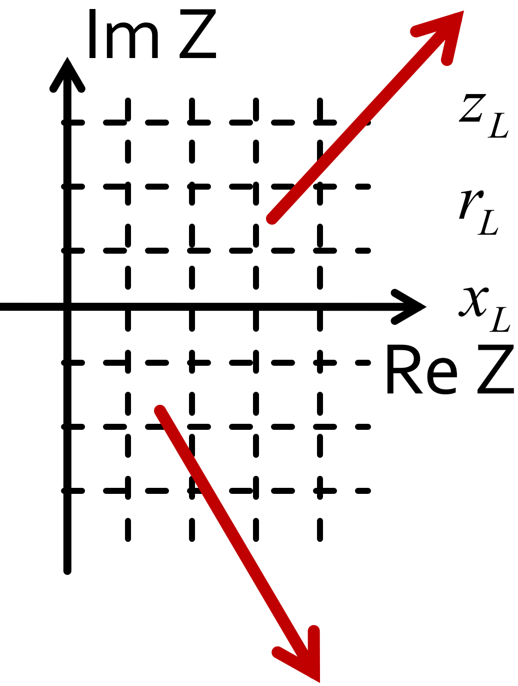


# The Smith Chart



# The Smith Chart

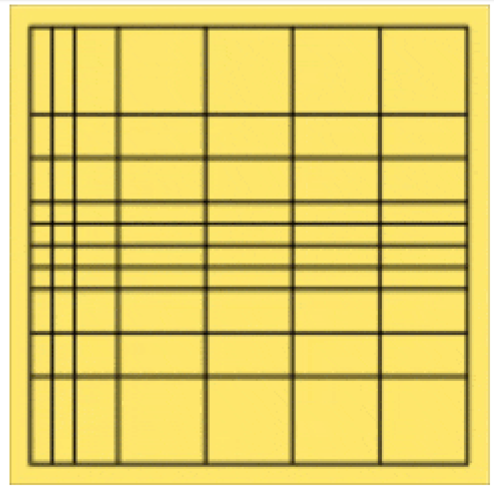
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

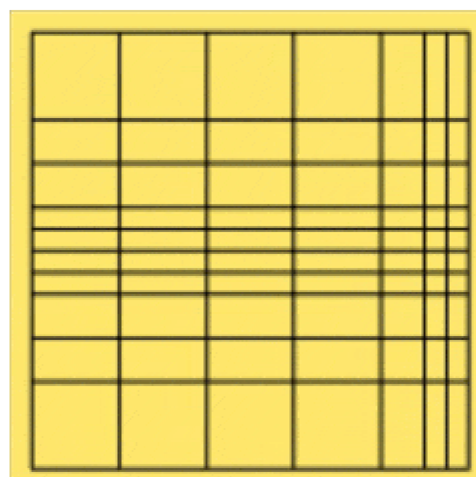


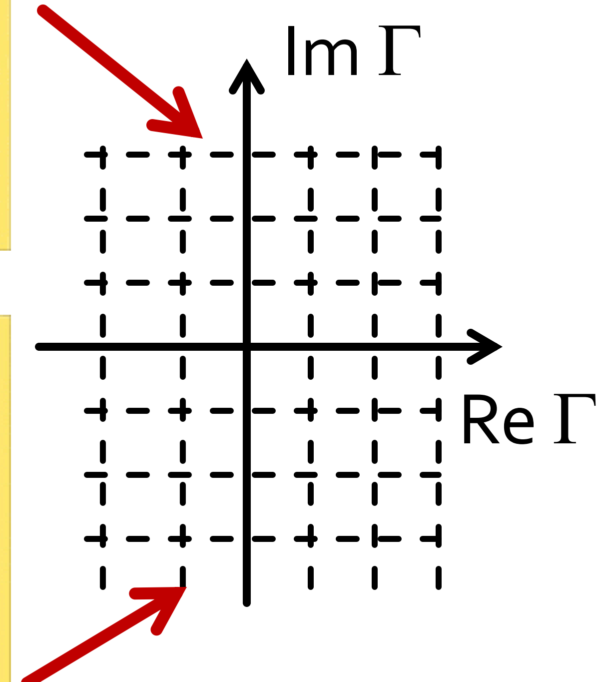

$$z_L = r_L + j \cdot x_L$$

$$r_L = ct.$$

$$x_L = ct.$$

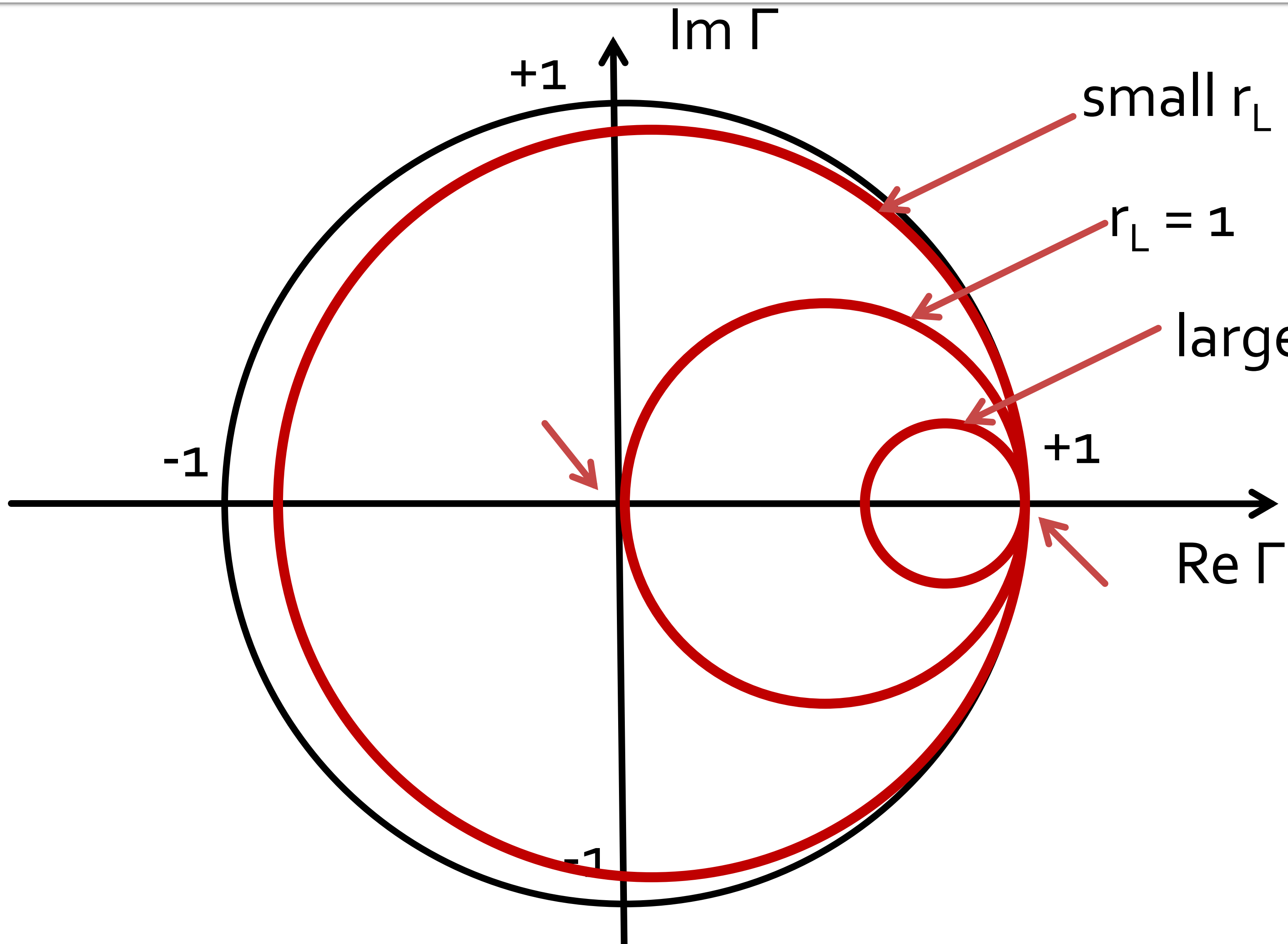




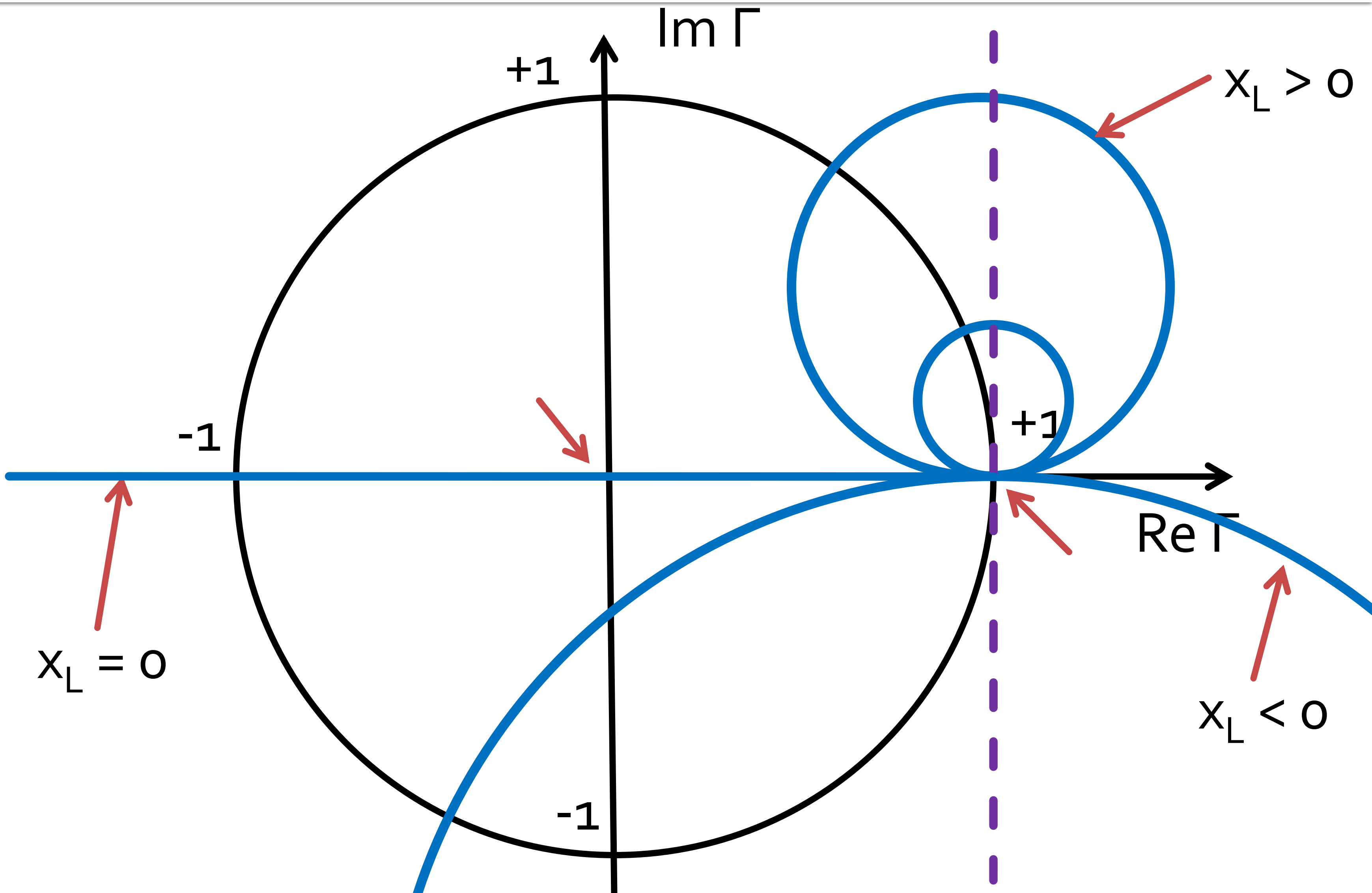



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y_L}{1 + y_L}$$

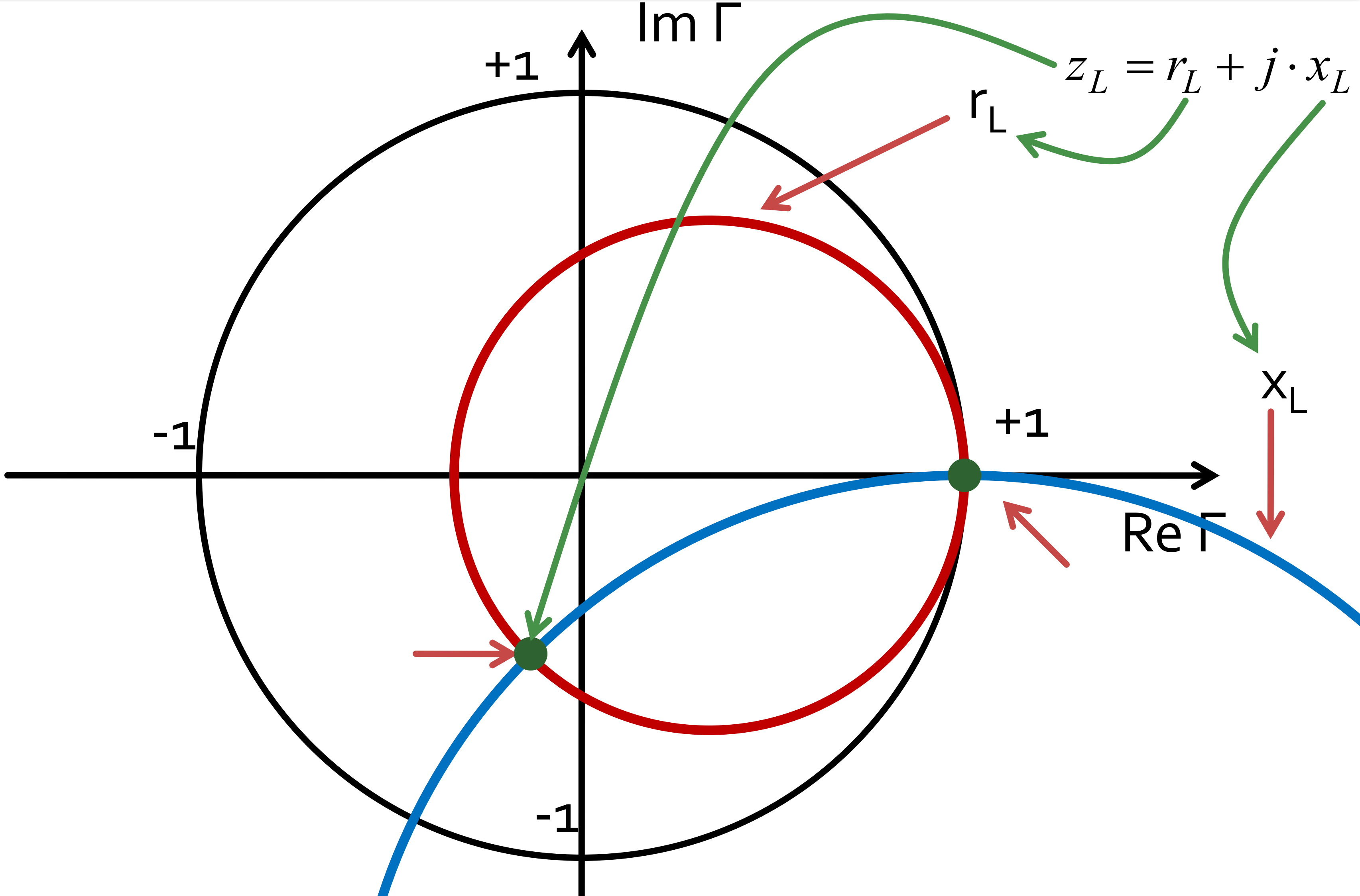
# The Smith Chart, resistance



# The Smith Chart, reactance



# The Smith Chart, impedance



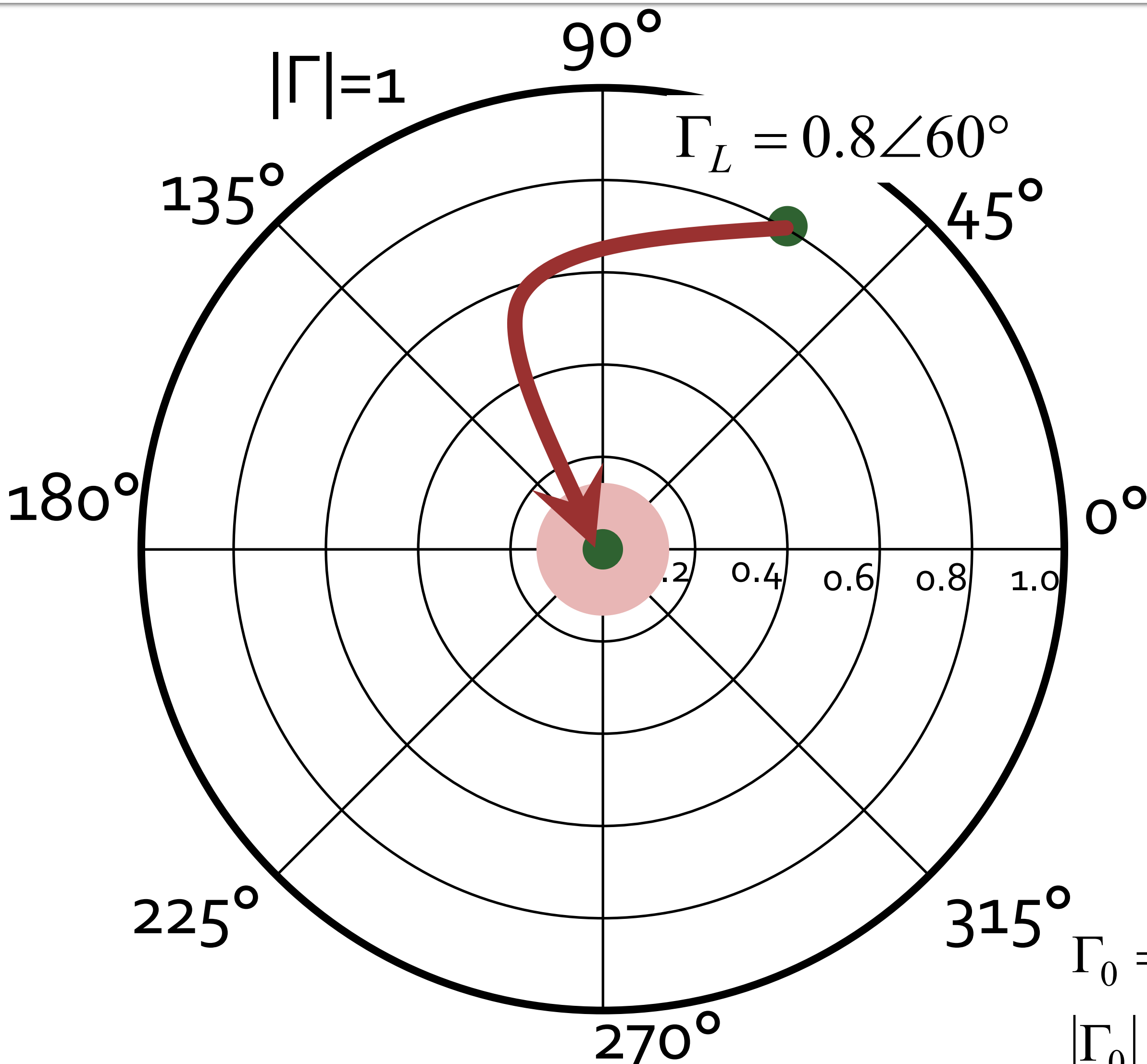
Impedance matching

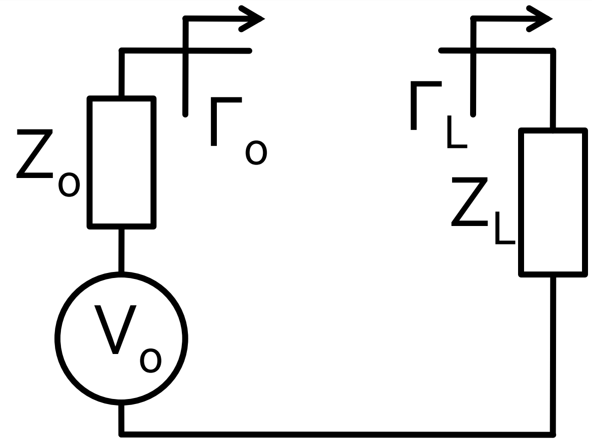
# Impedance Matching with lumped elements (L Networks)

# Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

# Impedance matching



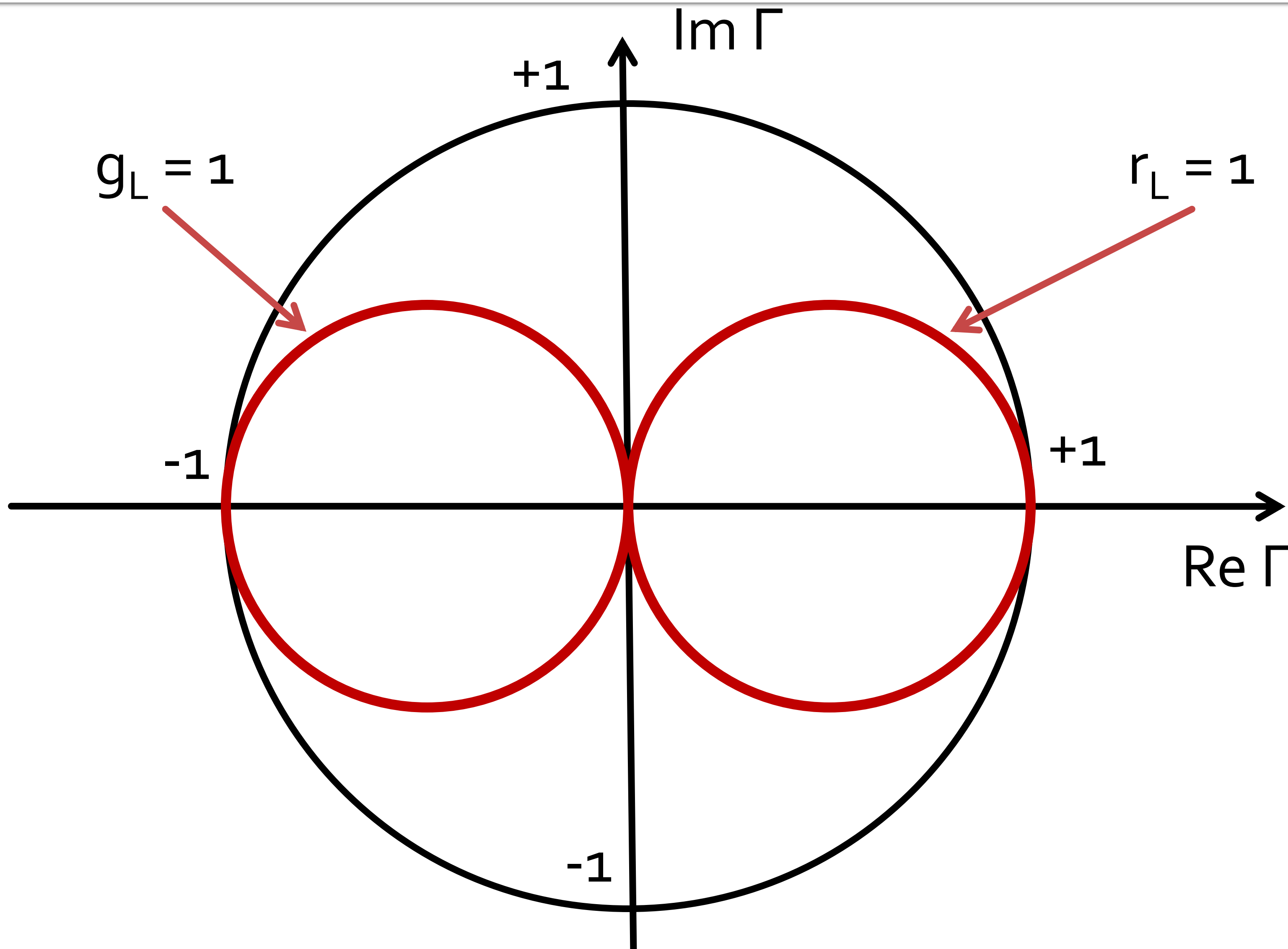




**How?**

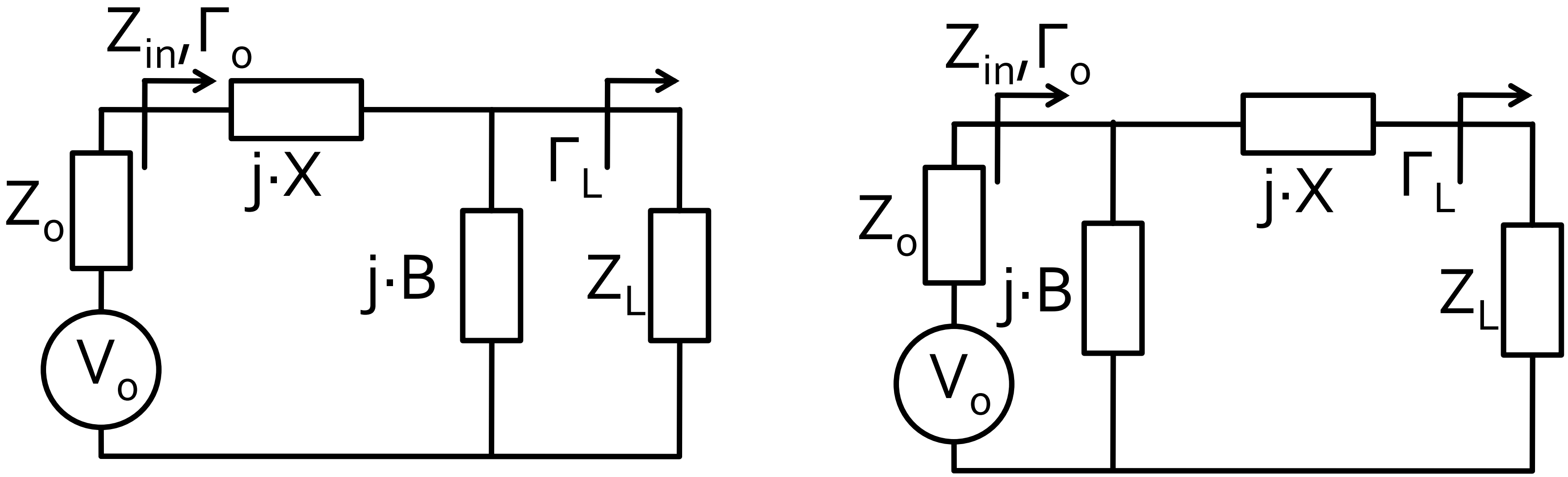
$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \leq \Gamma_m$ "good enough" match

# Smith chart, r=1 and g=1

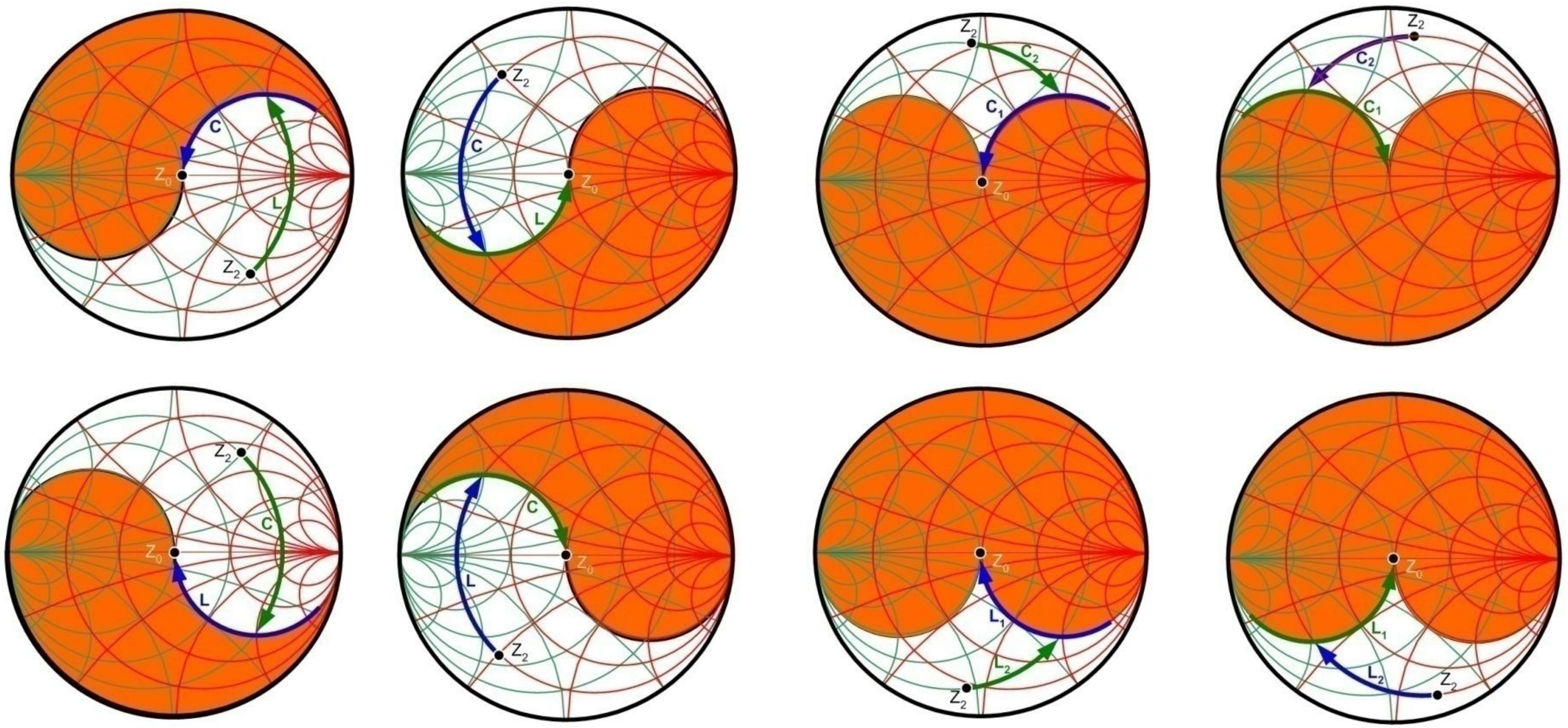


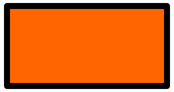
# Matching with 2 reactive elements (L Networks)




- Two steps matching
  - first reactive element moves the reflection coefficient **on the circle** $r_L = 1/g_L = 1$
  - second element compensates the remaining reactance and achieves the impedance match

# Matching with 2 reactive elements (L Networks)



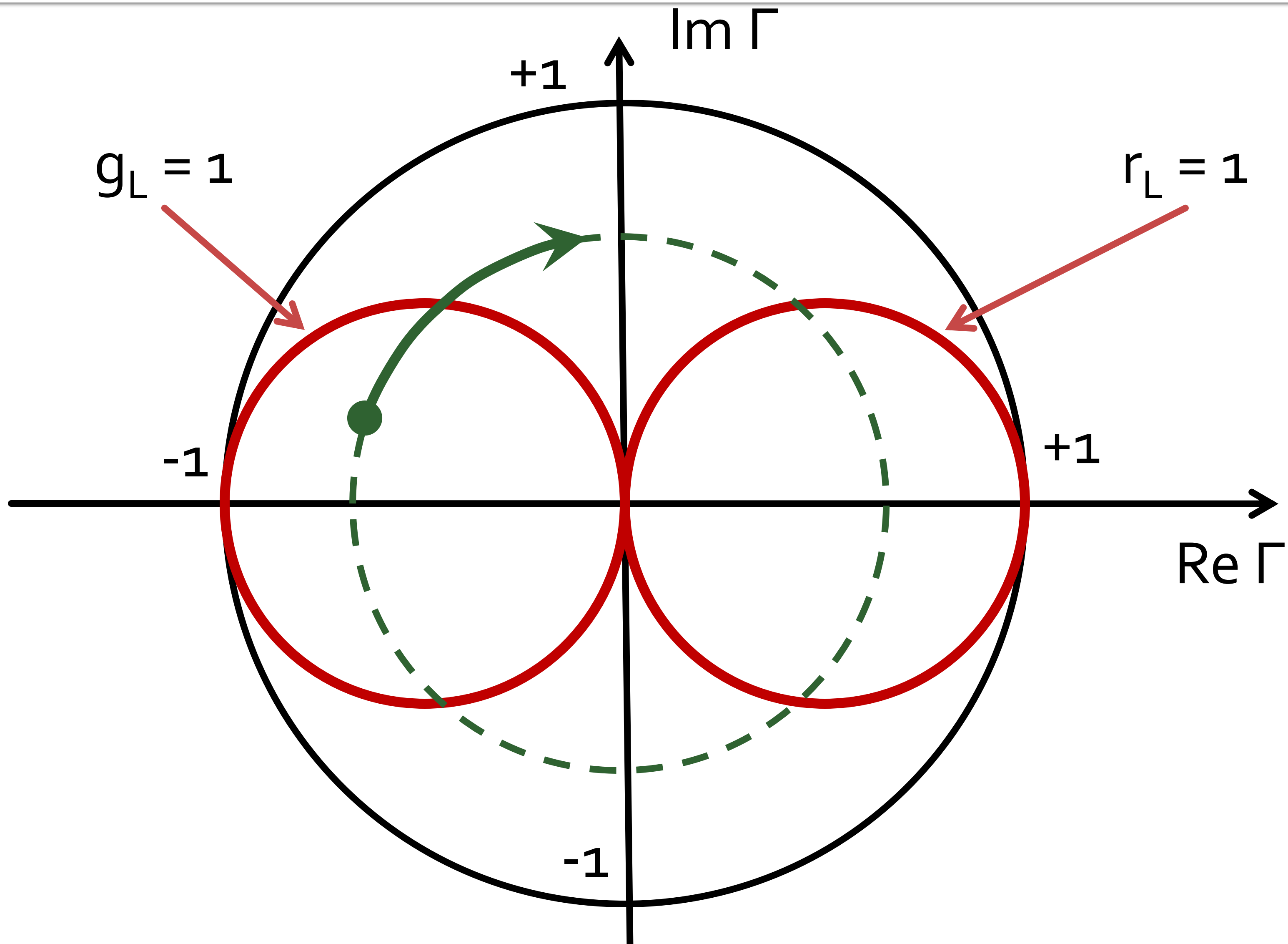



Forbidden area for current network

Impedance Matching

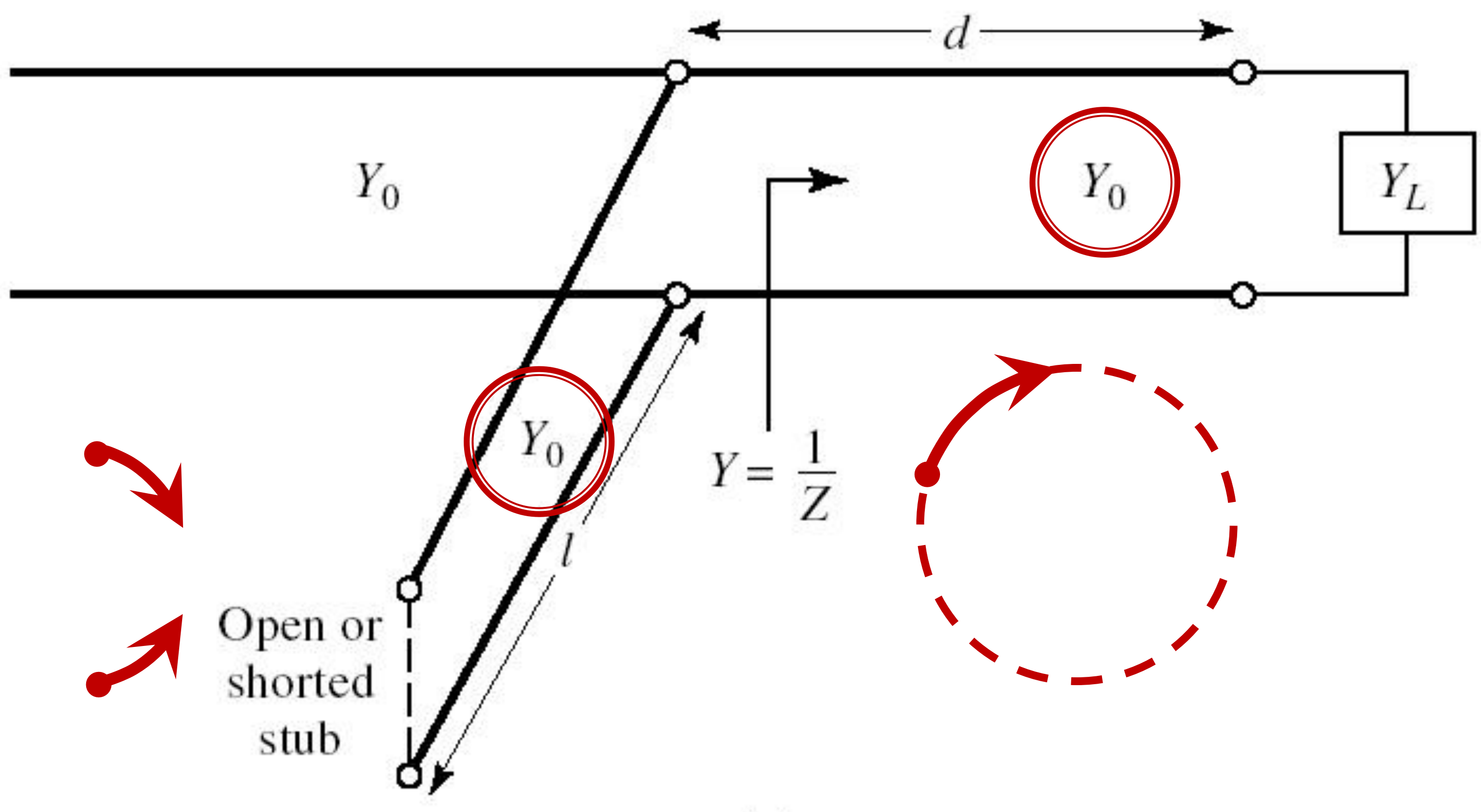
# Impedance Matching with Stubs

# Smith chart, r=1 and g=1



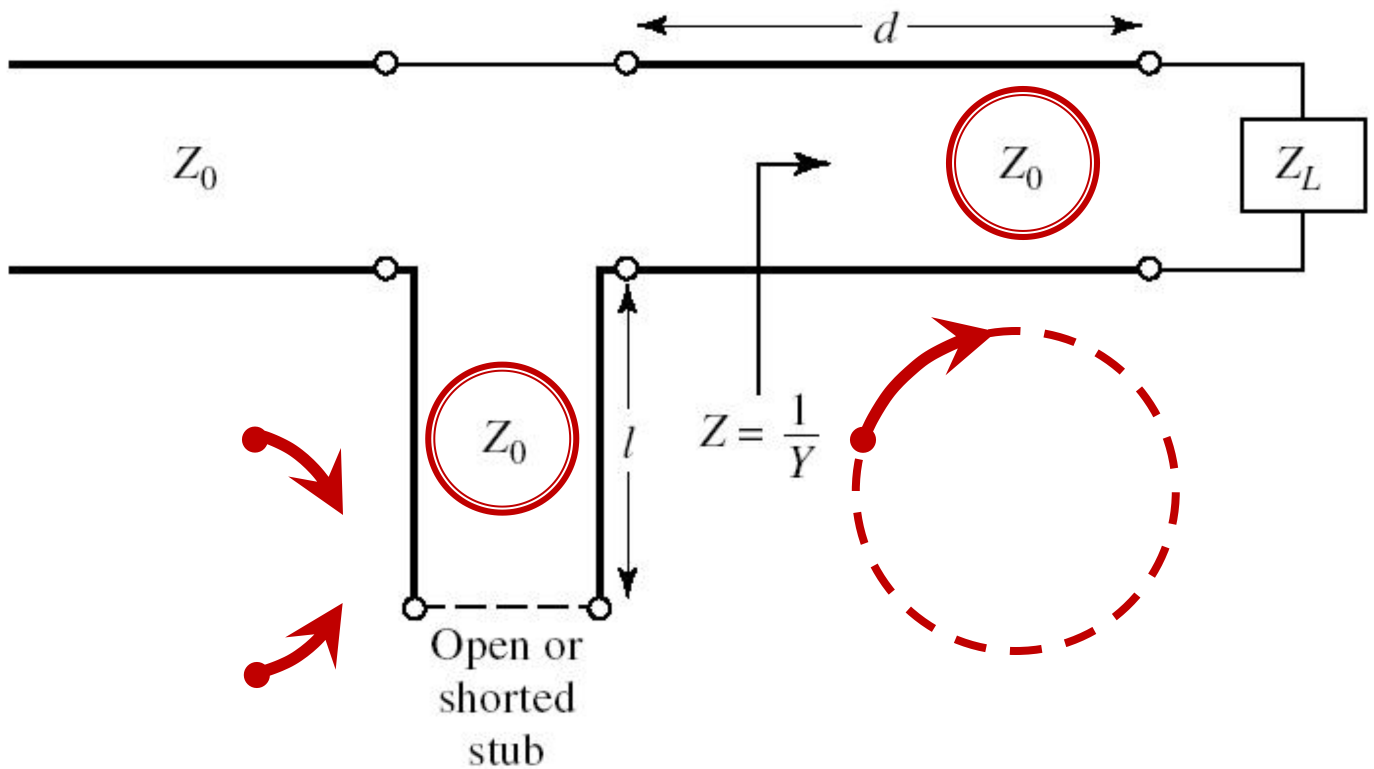
# Single stub tuning

- Shunt Stub



# Single stub tuning

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)

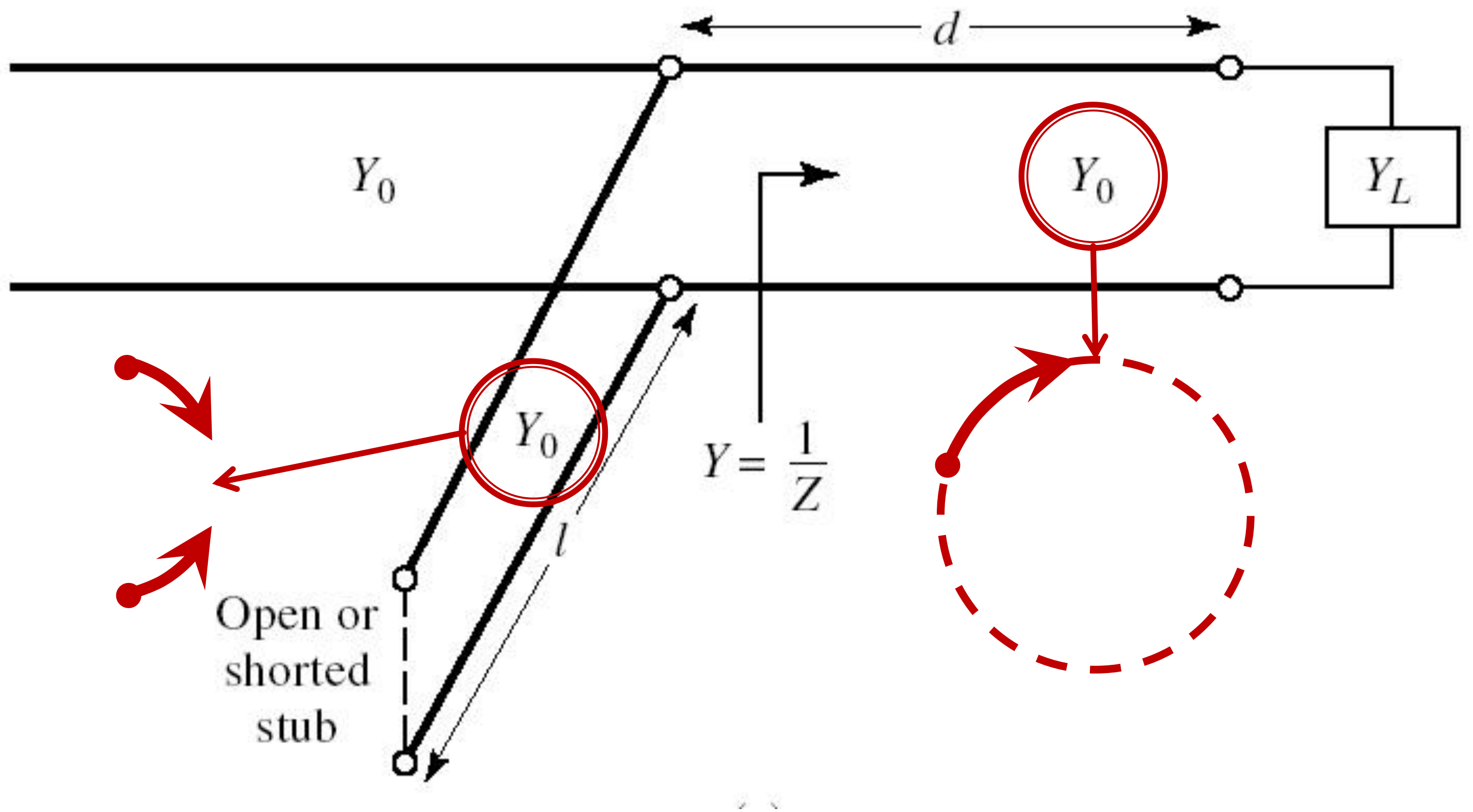


# Analytical solutions

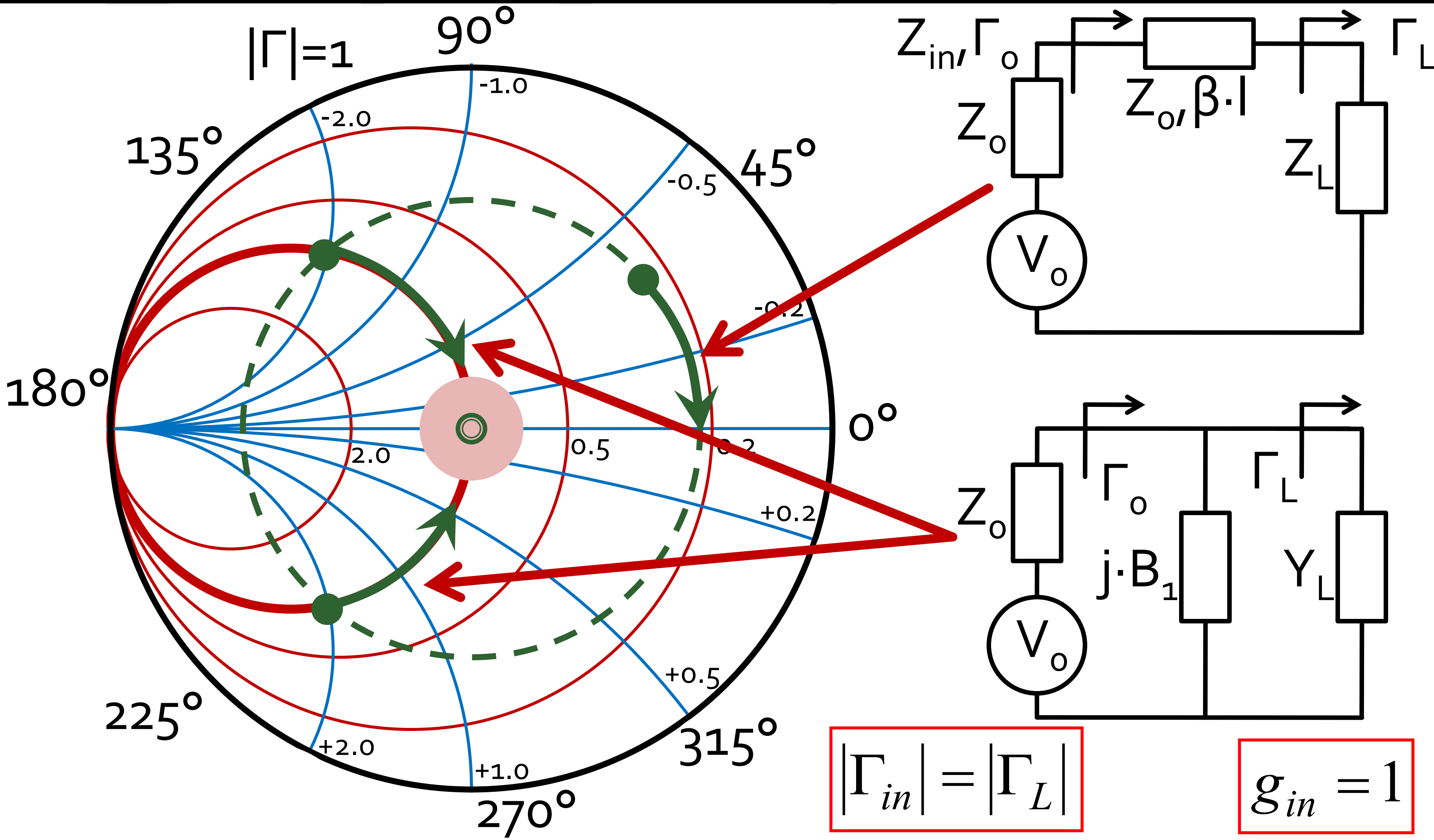
Exam / Project

# Case 1, Shunt Stub

- Shunt Stub



# Matching, series line + shunt susceptance




$$|\Gamma_{in}| = |\Gamma_L|$$

$$g_{in} = 1$$

# Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.593\angle 46.85°$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85° \qquad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

    $$(46.85° + 2\theta) = +126.35° \qquad \theta = +39.7° \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

    $$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -55.8°(+180°) \rightarrow \theta_{sp} = 124.2°$$

  - **"-" solution**

    $$(46.85° + 2\theta) = -126.35° \qquad \theta = -86.6°(+180°) \rightarrow \theta = 93.4°$$

    $$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 55.8°$$

# Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases} \quad \theta = \begin{cases} 39.7^\circ \\ 93.4^\circ \end{cases} \quad \mathrm{Im}[y_S(\theta)] = \begin{cases} -1.472 \\ +1.472 \end{cases} \quad \theta_{sp} = \begin{cases} -55.8^\circ + 180^\circ = 124.2^\circ \\ +55.8^\circ \end{cases}$$

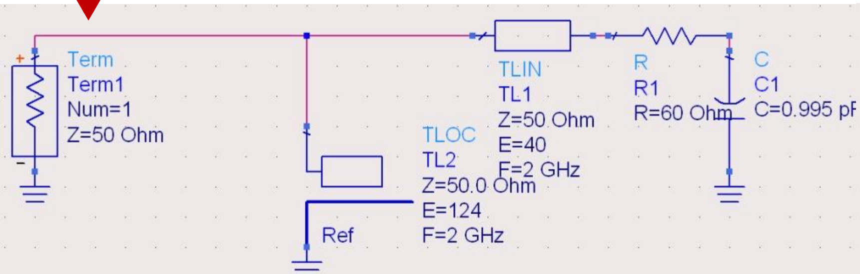
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

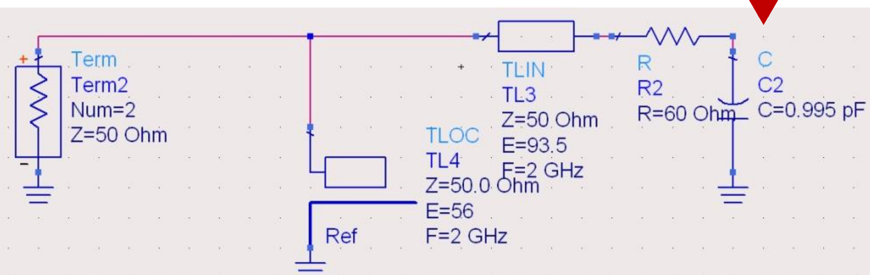
$$l_1 = \frac{39.7^\circ}{360^\circ} \cdot \lambda = 0.110 \cdot \lambda$$

$$l_2 = \frac{124.2^\circ}{360^\circ} \cdot \lambda = 0.345 \cdot \lambda$$

$$l_1 = \frac{93.4^\circ}{360^\circ} \cdot \lambda = 0.259 \cdot \lambda$$

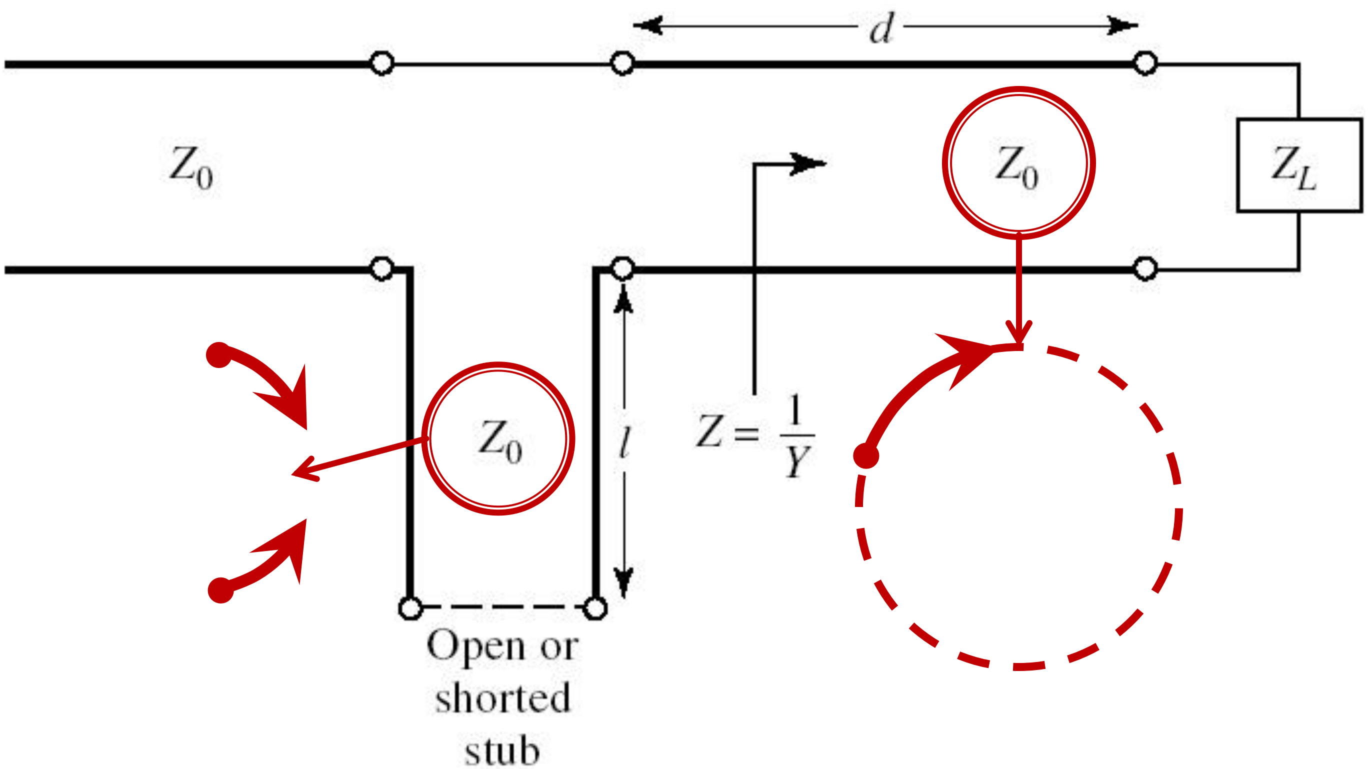
$$l_2 = \frac{55.8^\circ}{360^\circ} \cdot \lambda = 0.155 \cdot \lambda$$



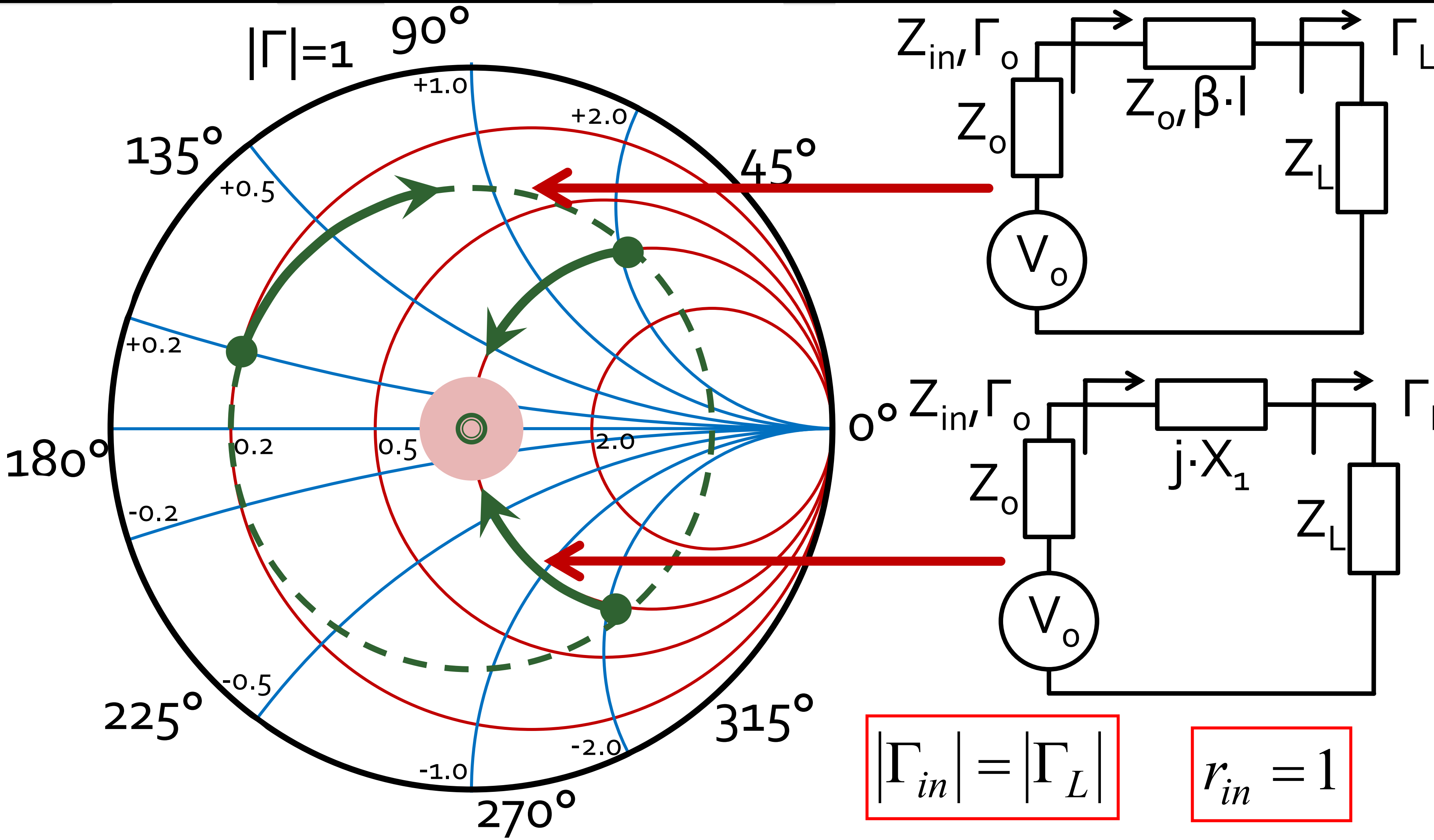



# Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



# Matching, series line + series reactance




$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$r_{in} = 1$$

# Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_S|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.555 \angle -29.92°$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92° \qquad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation
  - **"+" solution**

$$(-29.92° + 2\theta) = +56.28° \qquad \theta = 43.1° \qquad \operatorname{Im} z_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = -36.8°(+180°) \rightarrow \theta_{ss} = 143.2°$$

  - **"-" solution**

$$(-29.92° + 2\theta) = -56.28° \qquad \theta = -13.2°(+180°) \rightarrow \theta = 166.8°$$

$$\operatorname{Im} z_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.335 \qquad \theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = 36.8°$$

# Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +56.28° \\ -56.28° \end{cases} \quad \theta = \begin{cases} 43.1° \\ 166.8° \end{cases} \quad \mathrm{Im}[z_S(\theta)] = \begin{cases} +1.335 \\ -1.335 \end{cases} \quad \theta_{ss} = \begin{cases} -36.8° + 180° = 143.2° \\ +36.8° \end{cases}$$

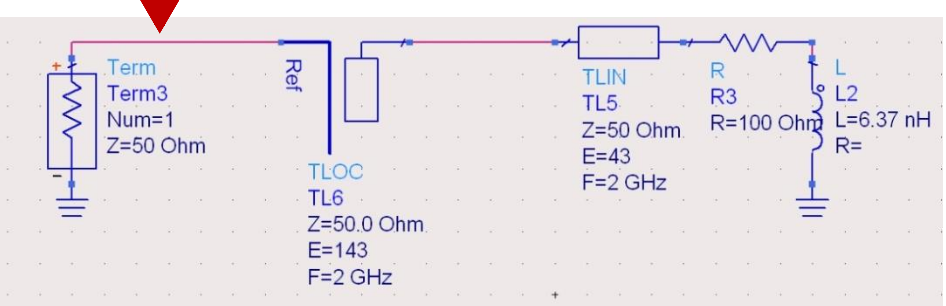
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

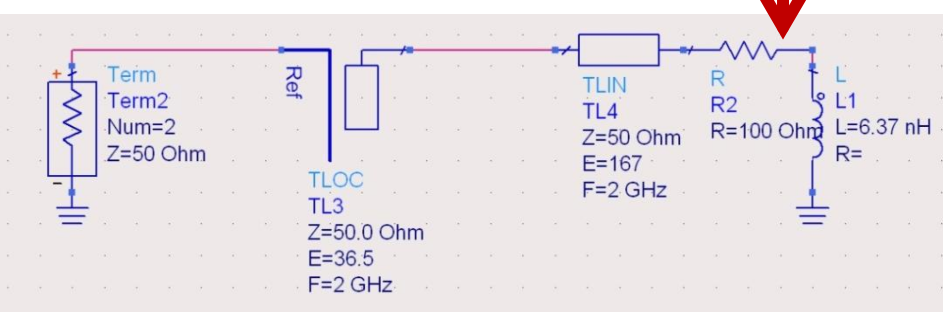
$$l_1 = \frac{43.1°}{360°} \cdot \lambda = 0.120 \cdot \lambda$$

$$l_2 = \frac{143.2°}{360°} \cdot \lambda = 0.398 \cdot \lambda$$

$$l_1 = \frac{166.8°}{360°} \cdot \lambda = 0.463 \cdot \lambda$$

$$l_2 = \frac{36.8°}{360°} \cdot \lambda = 0.102 \cdot \lambda$$



# Stub, observations

- adding or subtracting **180°** (λ/2) doesn't change the result (full rotation around the Smith Chart)

$$E = \beta \cdot l = \pi = 180^\circ \qquad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

  - if the lines/stubs result with **negative** "length"/ "electrical length" we add λ/2 / 180° to obtain physically realizable lines
- adding or subtracting **90°** (λ/4) change the stub impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \quad \Leftrightarrow \quad Z_{in,g} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

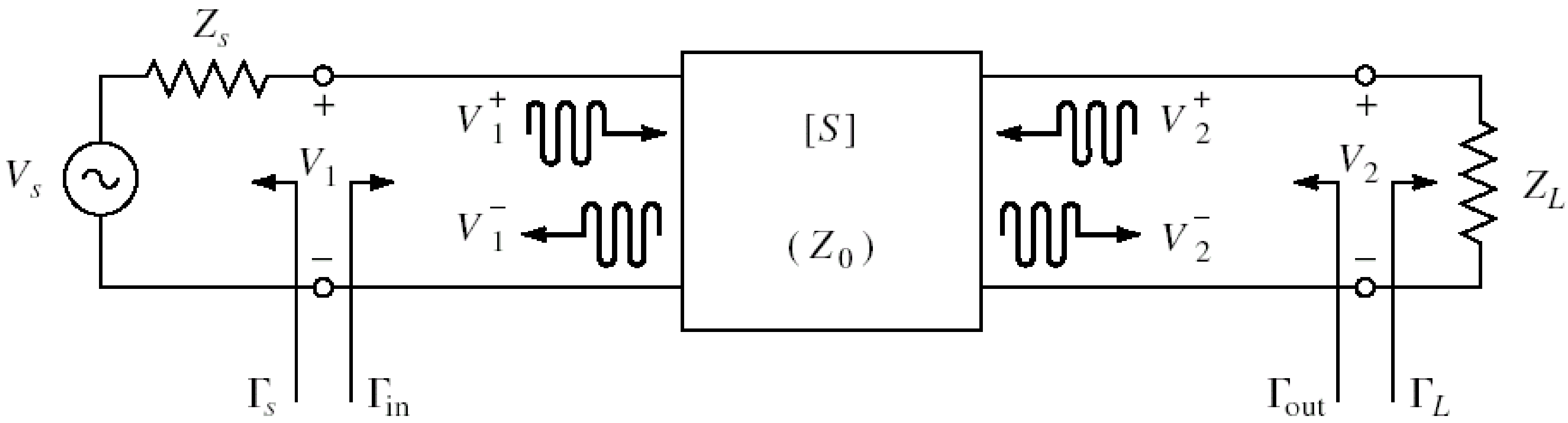
  - for the stub we can add or subtract 90° (λ/4) while in the same time changing **open-circuit** ⇔ **short-circuit**

# Microwave Amplifiers

# Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

# Amplifier as two-port




- Charaterized with S parameters
- normalized at Zo (implicit 50Ω)
- Datasheets: S parameters for specific bias conditions

# Datasheets

CEL

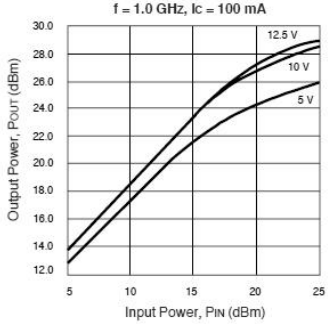
## NE46100 / NE46134

### NPN MEDIUM POWER MICROWAVE TRANSISTOR

#### FEATURES

- **HIGH DYNAMIC RANGE**
- **LOW IM DISTORTION:** -40 dBc
- **HIGH OUTPUT POWER :** 27.5 dBm at TYP
- **LOW NOISE:** 1.5 dB TYP at 500 MHz
- **LOW COST**

**NE46134**
**TYPICAL OUTPUT POWER**
**vs. INPUT POWER**
**f = 1.0 GHz, $I_C$ = 100 mA**




#### DESCRIPTION

The NE461 series of NPN silicon epitaxial bipolar transistors is designed for medium power applications requiring high dynamic range. This device exhibits an outstanding combination of high gain and low intermodulation distortion, as well as low noise figure. The NE461 series offers excellent performance and reliability at low cost through titanium, platinum, gold metallization system and direct nitride passivation of the surface of the chip. Devices are available in a low cost surface mount package (SOT-89) as well as in chip form.

#### ELECTRICAL CHARACTERISTICS ($T_A$ = 25°C)

| PART NUMBER<br>EIAJ[1] REGISTERED NUMBER<br>PACKAGE OUTLINE | | | NE46100<br><br>00 (CHIP) | | | NE46134<br>2SC4536<br>34 | | |
|---|---|---|---|---|---|---|---|---|
| SYMBOLS | PARAMETERS AND CONDITIONS | UNITS | MIN | TYP | MAX | MIN | TYP | MAX |
| $f_T$ | Gain Bandwidth Product at $V_{CE}$ = 10 V, $I_C$ = 100 mA | GHz | | 5.5 | | | 5.5 | |
| $NF_{MIN}$ | Minimum Noise Figure[3] at $V_{CE}$ = 10 V, $I_C$ = 50 mA, 500 MHz<br>$V_{CE}$ = 10 V, $I_C$ = 50 mA, 1 GHz | dB<br>dB | | 1.5<br>2.0 | | | 1.5<br>2.0 | |
| $G_L$ | Linear Gain, $V_{CE}$ = 12.5 V, $I_C$ = 100 mA, 2.0 GHz<br>$V_{CE}$ = 12.5 V, $I_C$ = 100 mA, 1.0 GHz | dB<br>dB | | 9.0 | | | <br>8.0 | |
| $\lvert S_{21E}\rvert^2$ | Insertion Power Gain at 10 V, 50 mA, f = 1.0 GHz | dB | | 10.0 | | 5.5 | 7.0 | |
| $h_{FE}$ | DC Current Gain[2] at $V_{CE}$ = 10 V, $I_C$ = 50 mA | | 40 | | 200 | 40 | | 200 |
| $I_{CBO}$ | Collector Cutoff Current at $V_{CB}$ = 20 V, $I_E$ = 0 mA . | µA | | | 5.0 | | | 5.0 |
| $I_{EBO}$ | Emitter Cutoff Current at $V_{EB}$ = 2 V, $I_C$ = 0 mA | µA | | | 5.0 | | | 5.0 |
| $P_{1dB}$ | Output Power at 1 dB Compression, $V_{CE}$ = 12.5 V, $I_C$ = 100 mA, 2.0 GHz<br>$V_{CE}$ = 12.5 V, $I_C$ = 100 mA, 1.0 GHz | dBm<br>dBm | 27.0 | | | | <br>27.5 | |
| $IM_3$ | Intermodulation Distortion, 10 V, 100 mA, F1 = 1.0 GHz, F2 = 0.99 GHz, | | | | | | | |

# Datasheets

**NE46100**

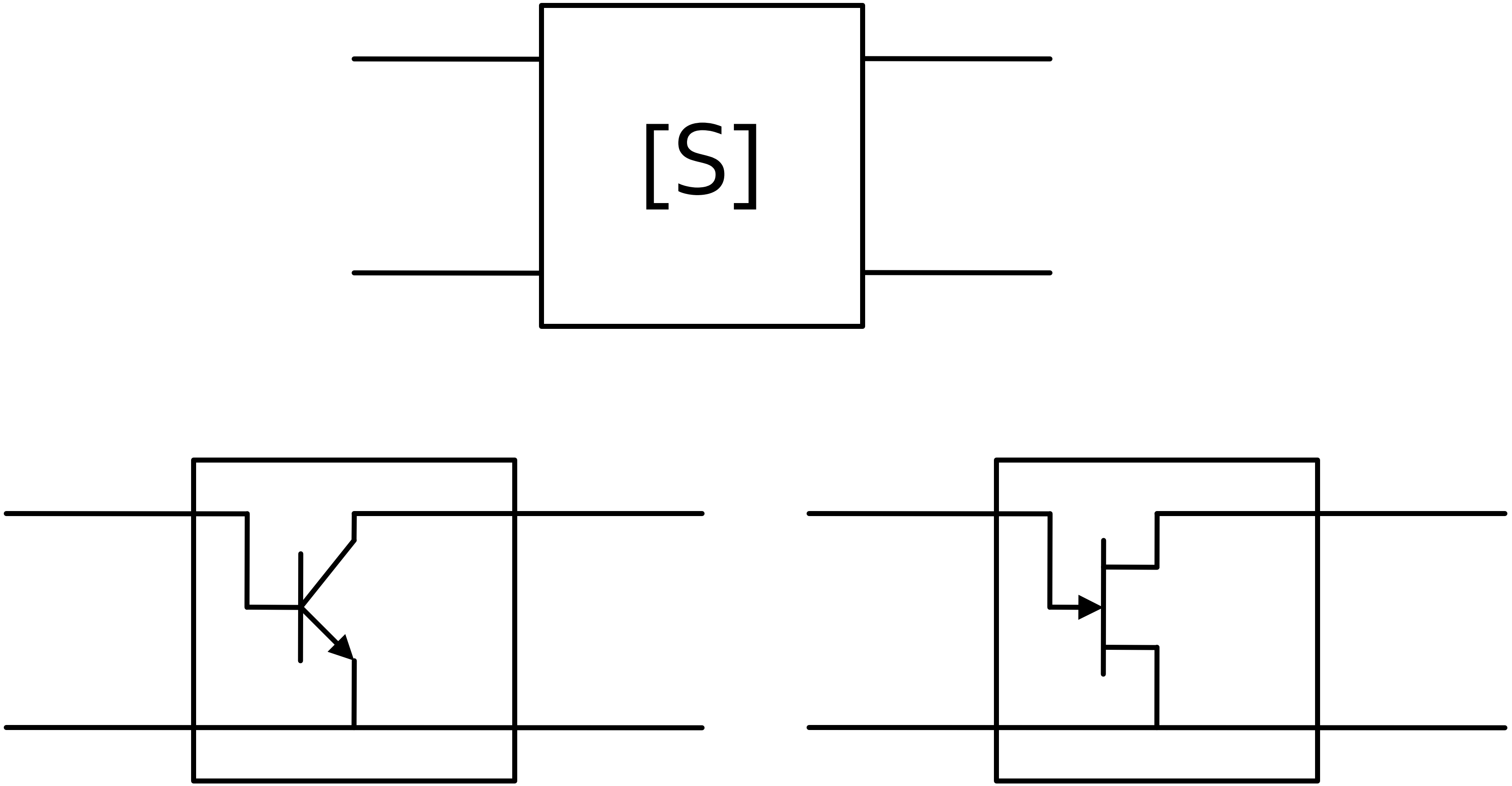
**$V_{CE}$ = 5 V, $I_C$ = 50 mA**

| FREQUENCY (MHz) | S11 MAG | S11 ANG | S21 MAG | S21 ANG | S12 MAG | S12 ANG | S22 MAG | S22 ANG | K | MAG[2] (dB) |
|---|---|---|---|---|---|---|---|---|---|---|
| 100 | 0.778 | -137 | 26.776 | 114 | 0.028 | 30 | 0.555 | -102 | 0.16 | 29.8 |
| 200 | 0.815 | -159 | 14.407 | 100 | 0.035 | 29 | 0.434 | -135 | 0.36 | 26.2 |
| 500 | 0.826 | -177 | 5.855 | 84 | 0.040 | 38 | 0.400 | -162 | 0.75 | 21.7 |
| 800 | 0.827 | 176 | 3.682 | 76 | 0.052 | 43 | 0.402 | -169 | 0.91 | 18.5 |
| 1000 | 0.826 | 173 | 2.963 | 71 | 0.058 | 47 | 0.405 | -172 | 1.02 | 16.3 |
| 1200 | 0.825 | 170 | 2.441 | 66 | 0.064 | 47 | 0.412 | -174 | 1.08 | 14.0 |
| 1400 | 0.820 | 167 | 2.111 | 61 | 0.069 | 47 | 0.413 | -176 | 1.17 | 12.4 |
| 1600 | 0.828 | 165 | 1.863 | 57 | 0.078 | 54 | 0.426 | -177 | 1.15 | 11.4 |
| 1800 | 0.827 | 162 | 1.671 | 53 | 0.087 | 50 | 0.432 | -178 | 1.14 | 10.6 |
| 2000 | 0.828 | 159 | 1.484 | 49 | 0.093 | 50 | 0.431 | -180 | 1.17 | 9.5 |
| 2500 | 0.822 | 153 | 1.218 | 39 | 0.11 | 48 | 0.462 | 177 | 1.18 | 7.8 |
| 3000 | 0.818 | 148 | 1.010 | 30 | 0.135 | 46 | 0.490 | 174 | 1.16 | 6.3 |
| 3500 | 0.824 | 142 | 0.876 | 21 | 0.147 | 44 | 0.507 | 170 | 1.16 | 5.3 |
| 4000 | 0.812 | 137 | 0.762 | 13 | 0.168 | 38 | 0.535 | 167 | 1.14 | 4.3 |

**$V_{CE}$ = 5 V, $I_C$ = 100 mA**

| FREQUENCY (MHz) | S11 MAG | S11 ANG | S21 MAG | S21 ANG | S12 MAG | S12 ANG | S22 MAG | S22 ANG | K | MAG[2] (dB) |
|---|---|---|---|---|---|---|---|---|---|---|
| 100 | 0.778 | -144 | 27.669 | 111 | 0.027 | 35 | 0.523 | -114 | 0.27 | 30.2 |
| 200 | 0.820 | -164 | 14.559 | 97 | 0.029 | 29 | 0.445 | -144 | 0.42 | 27.0 |
| 500 | 0.832 | -179 | 5.885 | 84 | 0.035 | 38 | 0.435 | -166 | 0.81 | 22.2 |
| 800 | 0.833 | 175 | 3.691 | 76 | 0.048 | 45 | 0.435 | -173 | 0.95 | 18.8 |
| 1000 | 0.831 | 172 | 2.980 | 71 | 0.056 | 51 | 0.437 | -176 | 1.05 | 16.0 |
| 1200 | 0.836 | 169 | 2.464 | 67 | 0.061 | 52 | 0.432 | -178 | 1.11 | 14.0 |
| 1400 | 0.829 | 166 | 2.121 | 61 | 0.072 | 53 | 0.447 | -180 | 1.12 | 12.6 |
| 1600 | 0.831 | 164 | 1.867 | 58 | 0.080 | 54 | 0.445 | 179 | 1.14 | 11.4 |

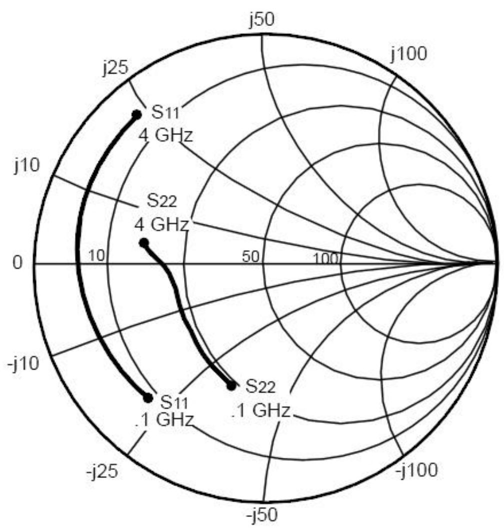
# S parameters for transistors

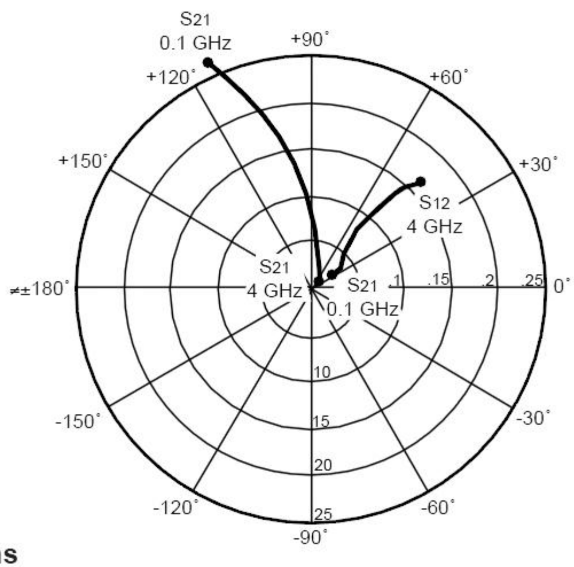


# Datasheets

**NE46100, NE46134**

**TYPICAL COMMON EMITTER SCATTERING PARAMETERS[1]** (TA = 25°C)



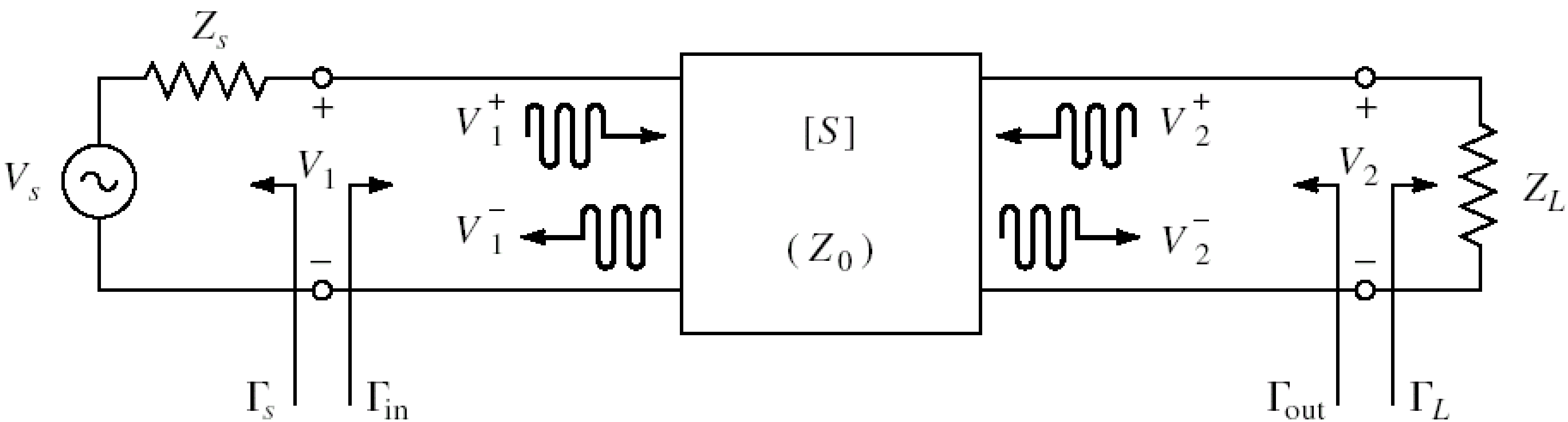

**Coordinates in Ohms**
**Frequency in GHz**
**VCE = 5 V, IC = 50 mA**

# S2P - Touchstone

- Touchstone file format (*.s2p)

```
! SIEMENS Small Signal Semiconductors
! VDS = 3.5 V     ID = 15 mA
# GHz  S  MA  R  50
! f        S11          S21          S12          S22
! GHz      MAG   ANG      MAG   ANG      MAG   ANG      MAG   ANG
 1.000 0.9800  -18.0   2.230  157.0  0.0240   74.0  0.6900  -15.0
 2.000 0.9500  -39.0   2.220  136.0  0.0450   57.0  0.6600  -30.0
 3.000 0.8900  -64.0   2.210  110.0  0.0680   40.0  0.6100  -45.0
 4.000 0.8200  -89.0   2.230   86.0  0.0850   23.0  0.5600  -62.0
 5.000 0.7400 -115.0   2.190   61.0  0.0990    7.0  0.4900  -80.0
 6.000 0.6500 -142.0   2.110   36.0  0.1070  -10.0  0.4100  -98.0
!
! f      Fmin   Gammaopt  rn/50
! GHz      dB    MAG  ANG    -
 2.000     1.00  0.72   27   0.84
 4.000     1.40  0.64   61   0.58
```

# Amplifier as two-port



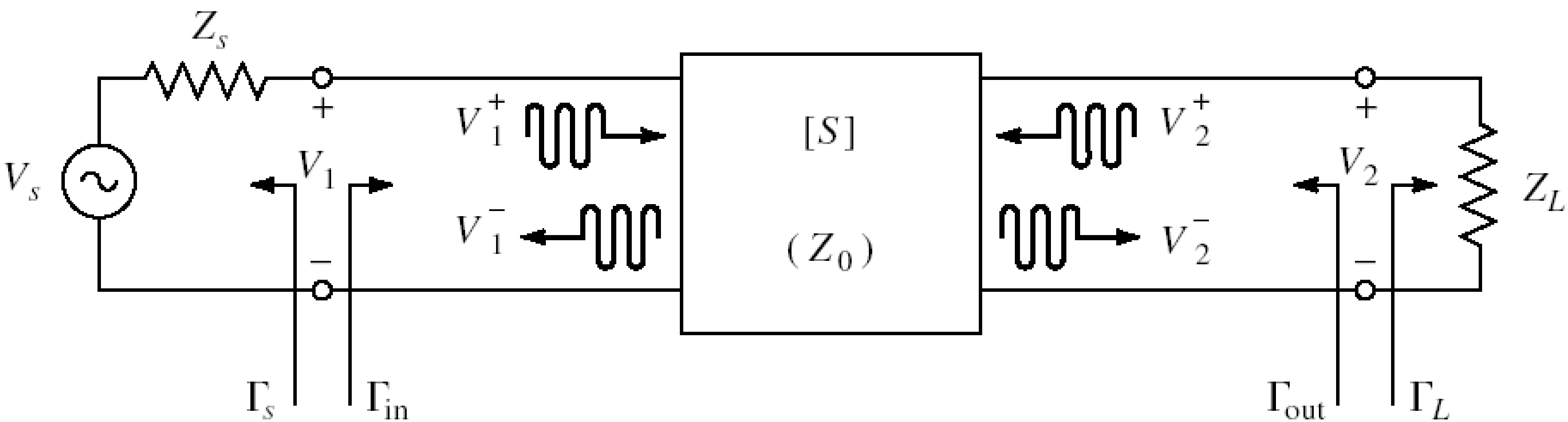

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \qquad \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$\Gamma_L = \frac{V_2^+}{V_2^-}$$

$$V_1^- = S_{11} \cdot V_1^+ + S_{12} \cdot V_2^+ = S_{11} \cdot V_1^+ + S_{12} \cdot \Gamma_L \cdot V_2^-$$

$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

# Amplifier as two-port




$$V_1^- = S_{11} \cdot V_1^+ + S_{12} \cdot V_2^+ = S_{11} \cdot V_1^+ + S_{12} \cdot \Gamma_L \cdot V_2^-$$

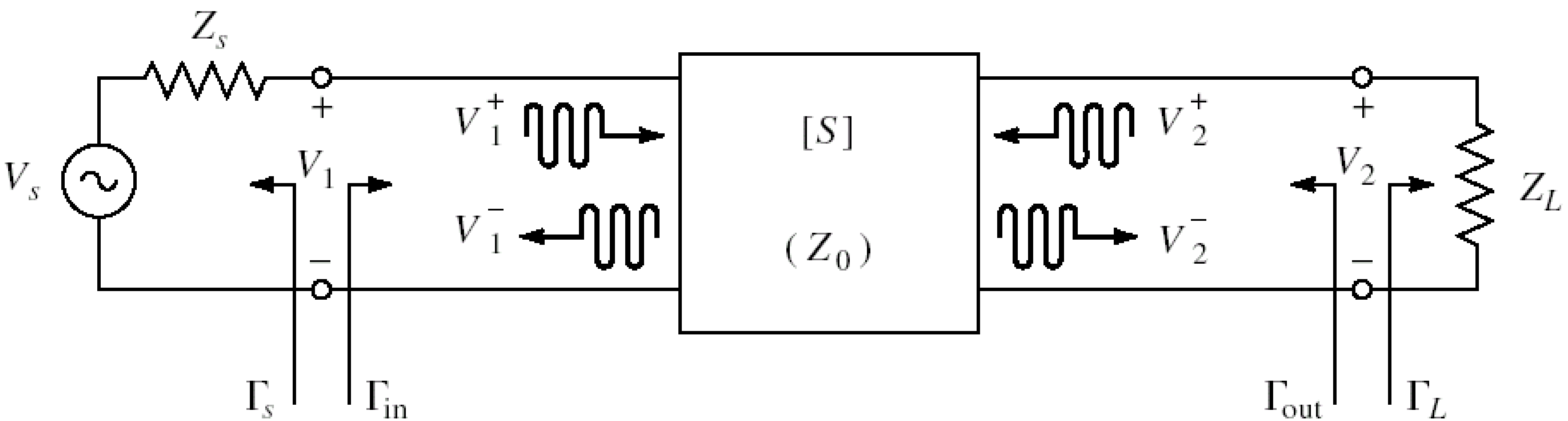
$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

- similarly

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

# Amplifier as two-port

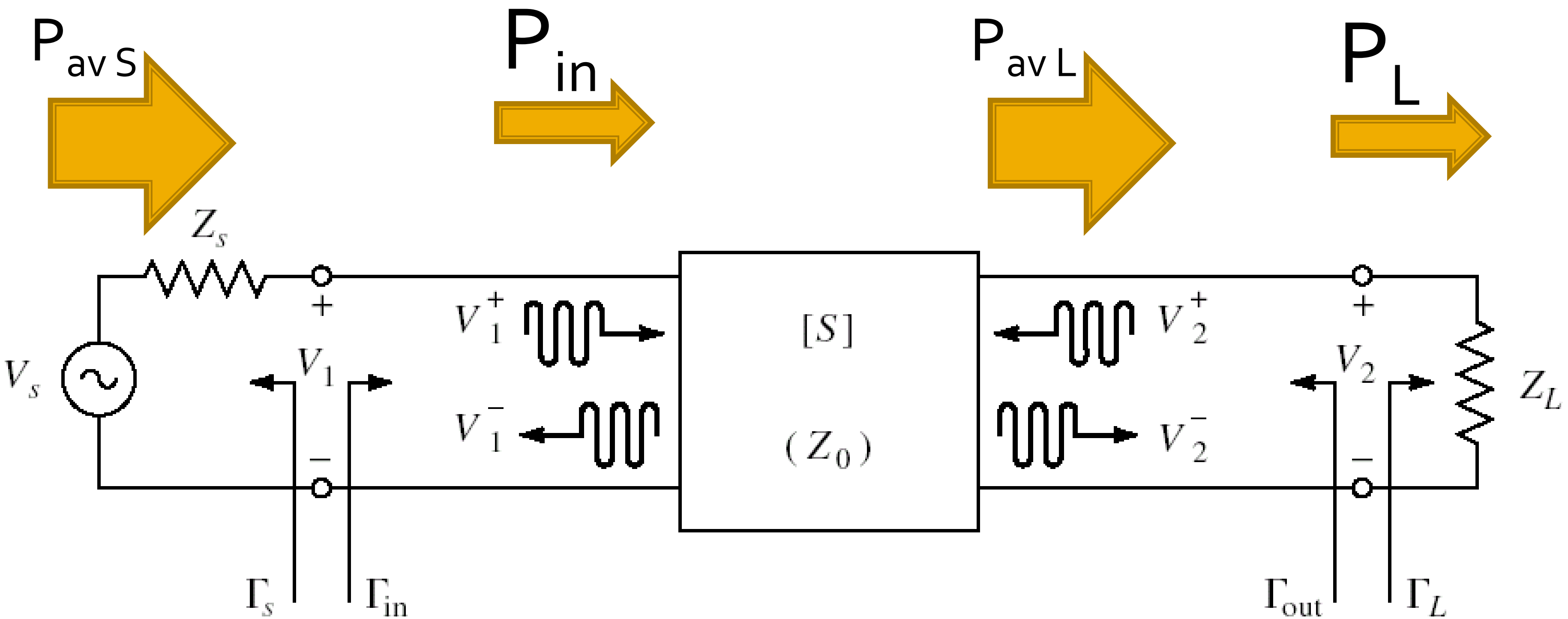



$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

# Power / Matching

- Two ports in which matching influences the power transfer



# Signal power

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$V_1 = \frac{V_S \cdot Z_{in}}{Z_S + Z_{in}} = V_1^+ + V_1^- = V_1^+ \cdot (1 + \Gamma_{in})$$

$$V_1^+ = \frac{V_S}{2} \frac{(1 - \Gamma_S)}{(1 - \Gamma_S \cdot \Gamma_{in})}$$

- L3 $P_{in} = \frac{1}{2 \cdot Z_0} \cdot \left|V_1^+\right|^2 \cdot \left(1 - \left|\Gamma_{in}\right|^2\right)$ $\quad P_L = \frac{1}{2 \cdot Z_0} \cdot \left|V_2^-\right|^2 \cdot \left(1 - \left|\Gamma_L\right|^2\right)$

$$P_{in} = \frac{\left|V_S\right|^2}{8 \cdot Z_0} \cdot \frac{\left|1 - \Gamma_S\right|^2}{\left|1 - \Gamma_S \cdot \Gamma_{in}\right|^2} \left(1 - \left|\Gamma_{in}\right|^2\right)$$

$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

$$V_2^- = \frac{S_{21} \cdot V_1^+}{1 - S_{22} \cdot \Gamma_L}$$

$$P_L = \frac{\left|V_1^+\right|^2}{2 \cdot Z_0} \cdot \frac{\left|S_{21}\right|^2}{\left|1 - S_{22} \cdot \Gamma_L\right|^2} \left(1 - \left|\Gamma_L\right|^2\right)$$

$$P_L = \frac{\left|V_S\right|^2}{8 \cdot Z_0} \cdot \frac{\left|S_{21}\right|^2 \cdot \left(1 - \left|\Gamma_L\right|^2\right)}{\left|1 - S_{22} \cdot \Gamma_L\right|^2} \cdot \frac{\left|1 - \Gamma_S\right|^2}{\left|1 - \Gamma_S \cdot \Gamma_{in}\right|^2}$$

# Signal power

- Signal power

$$P_{in} = \frac{|V_S|^2}{8\cdot Z_0}\cdot\frac{|1-\Gamma_S|^2}{|1-\Gamma_S\cdot\Gamma_{in}|^2}\left(1-|\Gamma_{in}|^2\right)$$

$$P_L = \frac{|V_S|^2}{8\cdot Z_0}\cdot\frac{|S_{21}|^2\cdot\left(1-|\Gamma_L|^2\right)}{|1-S_{22}\cdot\Gamma_L|^2}\cdot\frac{|1-\Gamma_S|^2}{|1-\Gamma_S\cdot\Gamma_{in}|^2}$$

- Power available from the source

$$P_{av\,S} = P_{in}\Big|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_S|^2}{8\cdot Z_0}\cdot\frac{|1-\Gamma_S|^2}{\left(1-|\Gamma_S|^2\right)}$$

- Power available on the load (from the network)

$$P_{av\,L} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_S|^2}{8\cdot Z_0}\cdot\frac{|S_{21}|^2\cdot|1-\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2\cdot\left(1-|\Gamma_{out}|^2\right)}$$

# Two-Port Power Gains

- Power Gain

$$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 \cdot \left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{in}|^2\right) \cdot |1 - S_{22} \cdot \Gamma_L|^2}$$

$$P_{in} = P_{in}(\Gamma_S, \Gamma_{in}(\Gamma_L), S)$$

$$P_L = P_L(\Gamma_S, \Gamma_{in}(\Gamma_L), S)$$

- The **actual** power gain **introduced** by the amplifier is less important because a higher gain may be accompanied by a **decrease** in input power (power actually drained from the source)
- We prefer to characterize the amplifier effect looking to the **power actually delivered to the load** in relation to the power **available from the source** (which is a constant)

# Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av\,L}}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)}{|1-S_{22}\cdot\Gamma_L|^2 \cdot \left(1-|\Gamma_{out}|^2\right)}$$

- **Transducer** power gain

$$G_T = \frac{P_L}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)\cdot\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\cdot\Gamma_{in}|^2 \cdot |1-S_{22}\cdot\Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

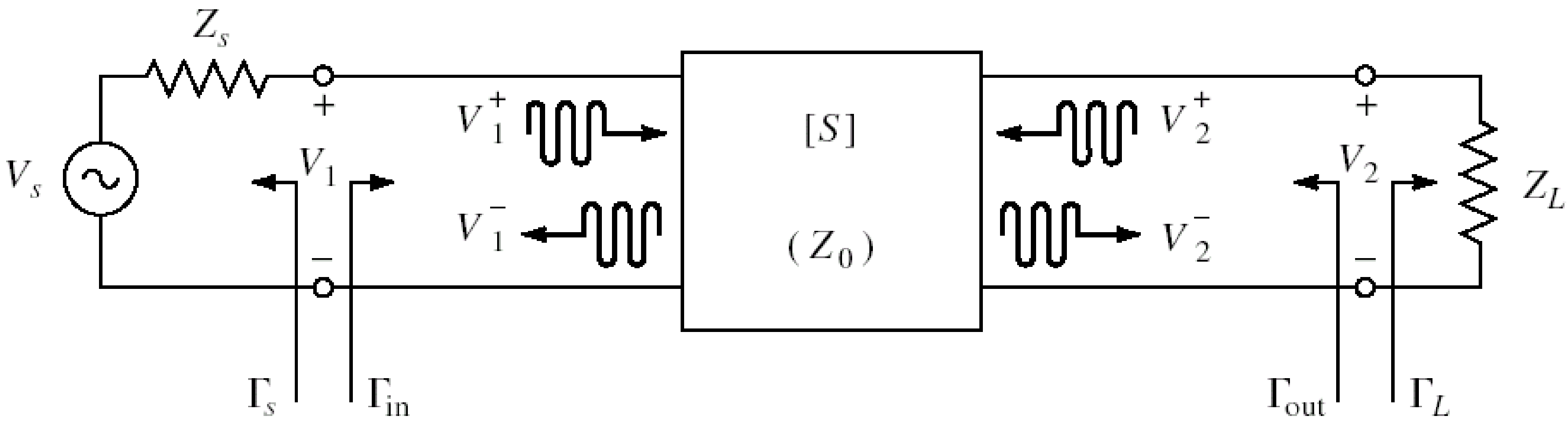
- **Unilateral transducer** power gain

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$S_{12} \cong 0 \qquad \Gamma_{in} = S_{11}$$

Input and output can be treated independently

# Amplifier as two-port

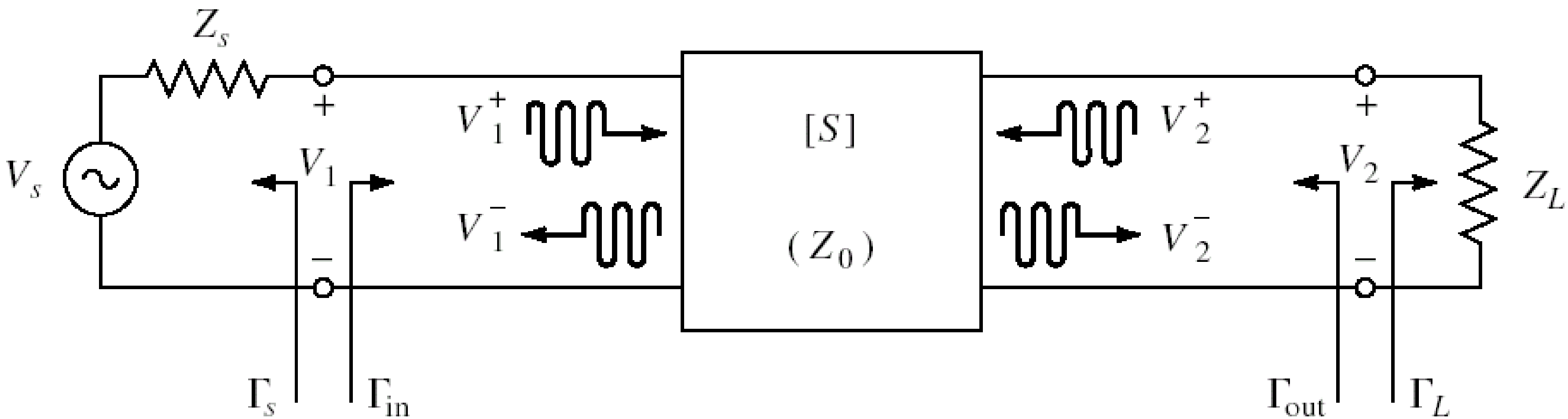



- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Microwave Amplifiers

# Stability

# Amplifier as two-port




- For an amplifier two-port we are interested in:
  - **stability**
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

# Stability

- L7 $\Gamma = \Gamma_r + j \cdot \Gamma_i$ $r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$

  $Z_{in}$ $\Gamma_{in} = \Gamma_r + j \cdot \Gamma_i$
- instability

  $\mathrm{Re}\{Z_{in}\} < 0 \quad \Leftrightarrow \quad 1 - \Gamma_r^2 - \Gamma_i^2 < 0 \qquad \Gamma_r^2 + \Gamma_i^2 > 1 \qquad |\Gamma_{in}| > 1$
- stability, $Z_{in}$
  - conditions to be met by $\Gamma_L$ to achieve (input) stability

    $|\Gamma_{in}| < 1$ $\left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$
- similarly $Z_{out}$
  - conditions to be met by $\Gamma_S$ to achieve (output) stability

# Stability

$$|\Gamma_{in}| < 1 \qquad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- We can calculate conditions to be met by $\Gamma_L$ to achieve stability

$$|\Gamma_{out}| < 1 \qquad \left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| < 1$$

- We can calculate conditions to be met by $\Gamma_S$ to achieve stability

# Stability

$$|\Gamma_{in}| < 1 \qquad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- The limit between stability/instability

$$|\Gamma_{in}| = 1 \qquad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| = 1$$

$$|S_{11} \cdot (1 - S_{22} \cdot \Gamma_L) + S_{12} \cdot S_{21} \cdot \Gamma_L| = |1 - S_{22} \cdot \Gamma_L|$$

- determinant of the S matrix $\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$

$$|S_{11} - \Delta \cdot \Gamma_L| = |1 - S_{22} \cdot \Gamma_L|$$

$$|S_{11} - \Delta \cdot \Gamma_L|^2 = |1 - S_{22} \cdot \Gamma_L|^2$$

# Stability

$$\left|S_{11}-\Delta\cdot\Gamma_L\right|^2=\left|1-S_{22}\cdot\Gamma_L\right|^2$$

$$a\cdot a^*=|a|\cdot e^{j\theta}\cdot|a|\cdot e^{-j\theta}=|a|^2$$

$$|a+b|^2=(a+b)\cdot(a+b)^*=(a+b)\cdot\left(a^*+b^*\right)=|a|^2+|b|^2+a^*\cdot b+a\cdot b^*$$

$$|S_{11}|^2+|\Delta|^2\cdot|\Gamma_L|^2-\left(\Delta\cdot\Gamma_L\cdot S_{11}^*+\Delta^*\cdot\Gamma_L^*\cdot S_{11}\right)=1+|S_{22}|^2\cdot|\Gamma_L|^2-\left(S_{22}^*\cdot\Gamma_L^*+S_{22}\cdot\Gamma_L\right)$$

$$\left(|S_{22}|^2-|\Delta|^2\right)\cdot\Gamma_L\cdot\Gamma_L^*-\left(S_{22}-\Delta\cdot S_{11}^*\right)\cdot\Gamma_L-\left(S_{22}^*-\Delta^*\cdot S_{11}\right)\cdot\Gamma_L^*=|S_{11}|^2-1$$

$$\Gamma_L\cdot\Gamma_L^*-\frac{\left(S_{22}-\Delta\cdot S_{11}^*\right)\cdot\Gamma_L+\left(S_{22}^*-\Delta^*\cdot S_{11}\right)\cdot\Gamma_L^*}{|S_{22}|^2-|\Delta|^2}=\frac{|S_{11}|^2-1}{|S_{22}|^2-|\Delta|^2}\quad\Big/+\frac{\left|S_{22}-\Delta\cdot S_{11}^*\right|^2}{\left(|S_{22}|^2-|\Delta|^2\right)^2}$$

$$\left|\Gamma_L-\frac{\left(S_{22}-\Delta\cdot S_{11}^*\right)^*}{|S_{22}|^2-|\Delta|^2}\right|^2=\frac{|S_{11}|^2-1}{|S_{22}|^2-|\Delta|^2}+\frac{\left|S_{22}-\Delta\cdot S_{11}^*\right|^2}{\left(|S_{22}|^2-|\Delta|^2\right)^2}$$

# Plan complex – geometrie analitica

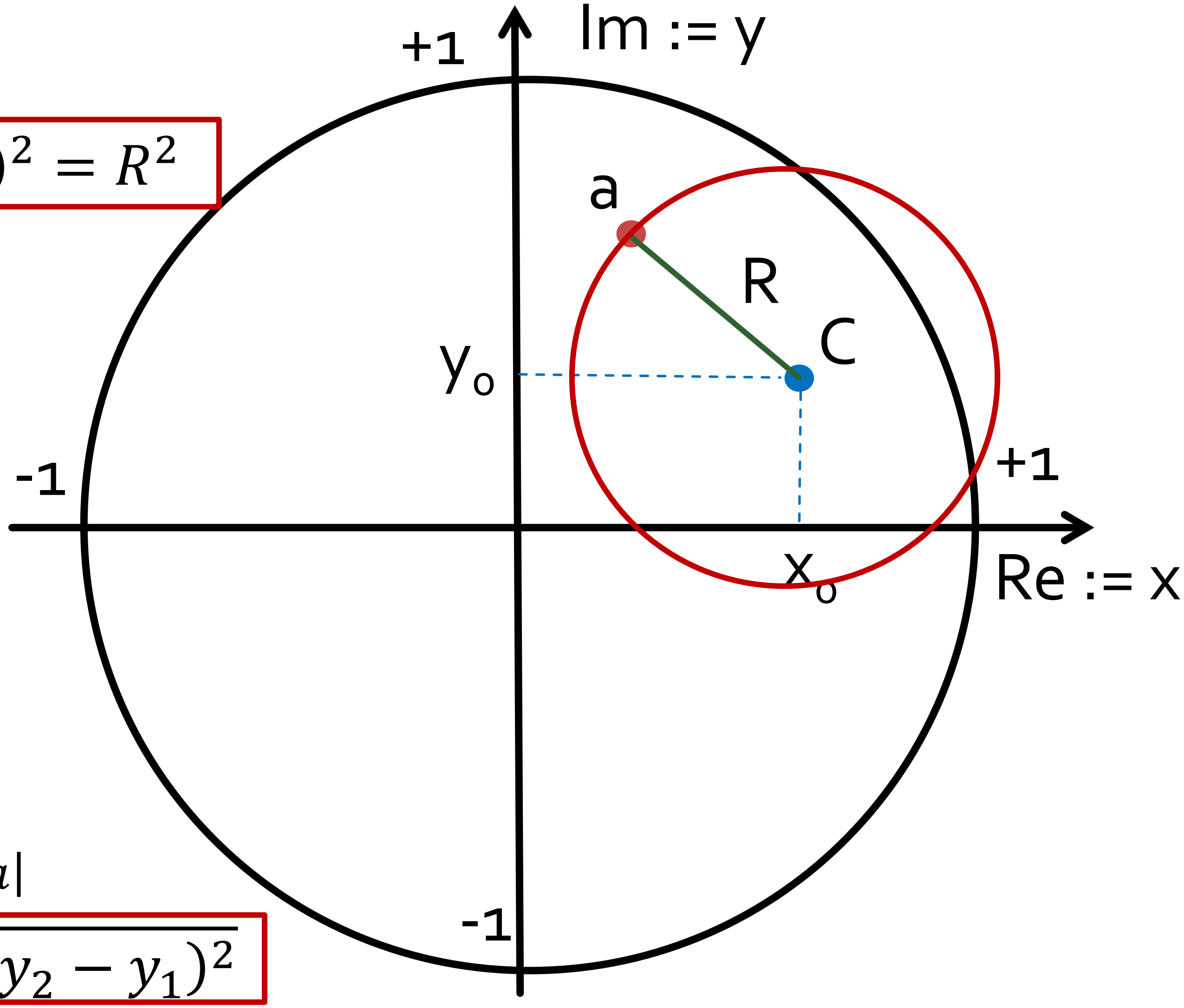
- solutii

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$a = x + j \cdot y$$

$$C = x_0 + j \cdot y_0$$

$$D(a, C) = |a - C| = R$$




$$a = x_1 + j \cdot y_1$$

$$b = x_2 + j \cdot y_2$$

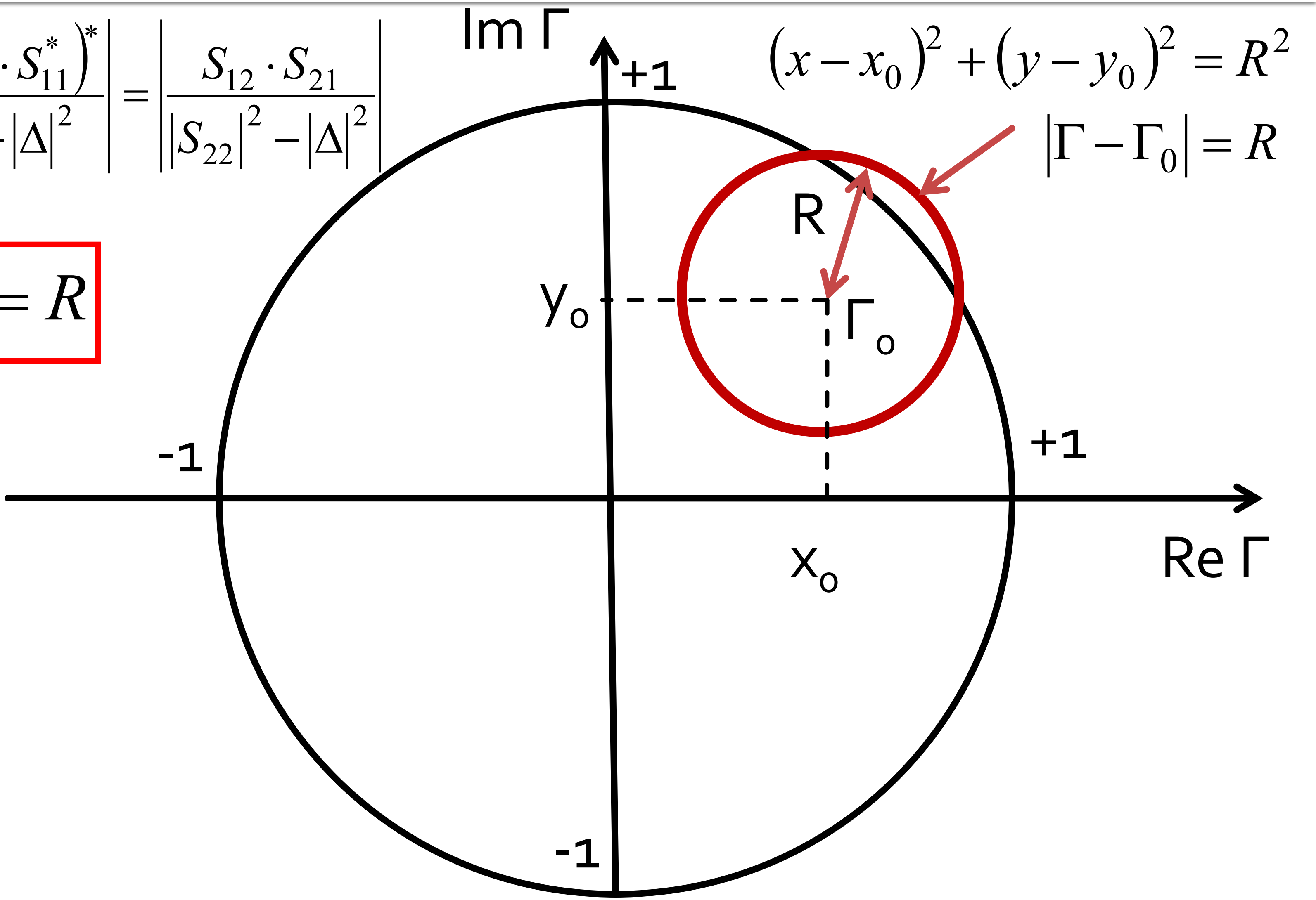
$$D(a, b) = |a - b| = |b - a|$$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

# Stability

$$\left|\Gamma_L - \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12} \cdot S_{21}}{|S_{22}|^2 - |\Delta|^2}\right|$$

$$|\Gamma - \Gamma_0| = R$$



# Output stability circle (CSOUT)

$$\left|\Gamma_L - \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12} \cdot S_{21}}{|S_{22}|^2 - |\Delta|^2}\right| \qquad |\Gamma_L - C_L| = R_L$$

- We obtain the equation of a circle in the complex plane, which represents the locus of $\Gamma_L$ for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- This circle is the **output stability circle** ($\Gamma_L$)

$$C_L = \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} \qquad R_L = \frac{|S_{12} \cdot S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|}$$

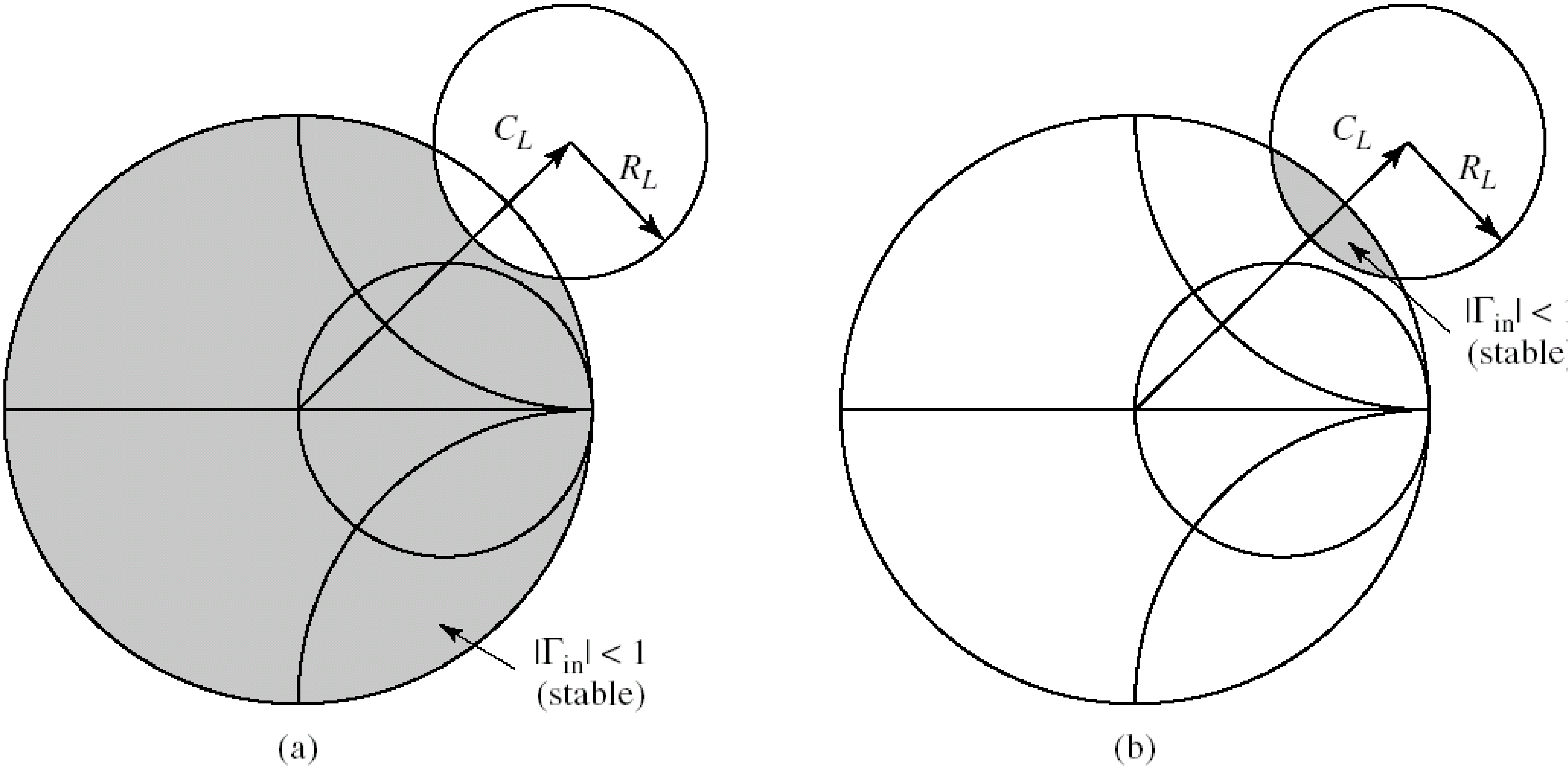
# Input stability circle (CSIN)

- Similarly
  $$|\Gamma_{out}| = 1 \qquad \left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| = 1$$
- We obtain the equation of a circle in the complex plane, which represents the locus of $\Gamma_S$ for the **limit between stability and instability** ($|\Gamma_{out}| = 1$)
- This circle is the **input stability circle** ($\Gamma_S$)
  $$C_S = \frac{\left(S_{11} - \Delta \cdot S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} \qquad R_S = \frac{|S_{12} \cdot S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|}$$

# Output stability circle (CSOUT)

- The **output stability circle** represents the locus of $\Gamma_L$ for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- The circle divides the complex planes in two areas, the **inside** and the **outside** of the circle
- The two areas will represent the locus of $\Gamma_L$ for stability ($|\Gamma_{in}|<1$) / instability ($|\Gamma_{in}|>1$)

# Output stability circle (CSOUT)




- Two cases possible: (a) stable outside/ (b) stable inside

# Output stability circle (CSOUT)

- Identification of the stability / instability regions
  - The center of the Smith Chart in $\Gamma_L$ complex plane corresponds to $\Gamma_L = 0$
  - Input reflection coefficient

$$\Gamma_{in} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \qquad \Gamma_{in}\big|_{\Gamma_L = 0} = S_{11} \qquad \left|\Gamma_{in}\right|_{\Gamma_L = 0} = \left|S_{11}\right|$$

  - A decision can be made based on |S11| value and on the position of the center of the Smith chart (origin of the complex plane) relative to the circle

# Identification of the stability / instability regions

- Output stability circle
  - **|S11| < 1** → the center of the Smith chart on which $\Gamma_L$ is represented is a **stable point**, so it's placed in the stability region (most often situation)
  - |S11| > 1 → the center of the Smith chart on which $\Gamma_L$ is represented is an **unstable point**, so it's placed in the instability region
- Input stability circle
  - **|S22| < 1** → the center of the Smith chart on which $\Gamma_S$ is represented is a **stable point**, so it's placed in the stability region (most often situation)
  - |S22| > 1 → the center of the Smith chart on which $\Gamma_S$ is represented is a **unstable point**, so it's placed in the instability region

# Example

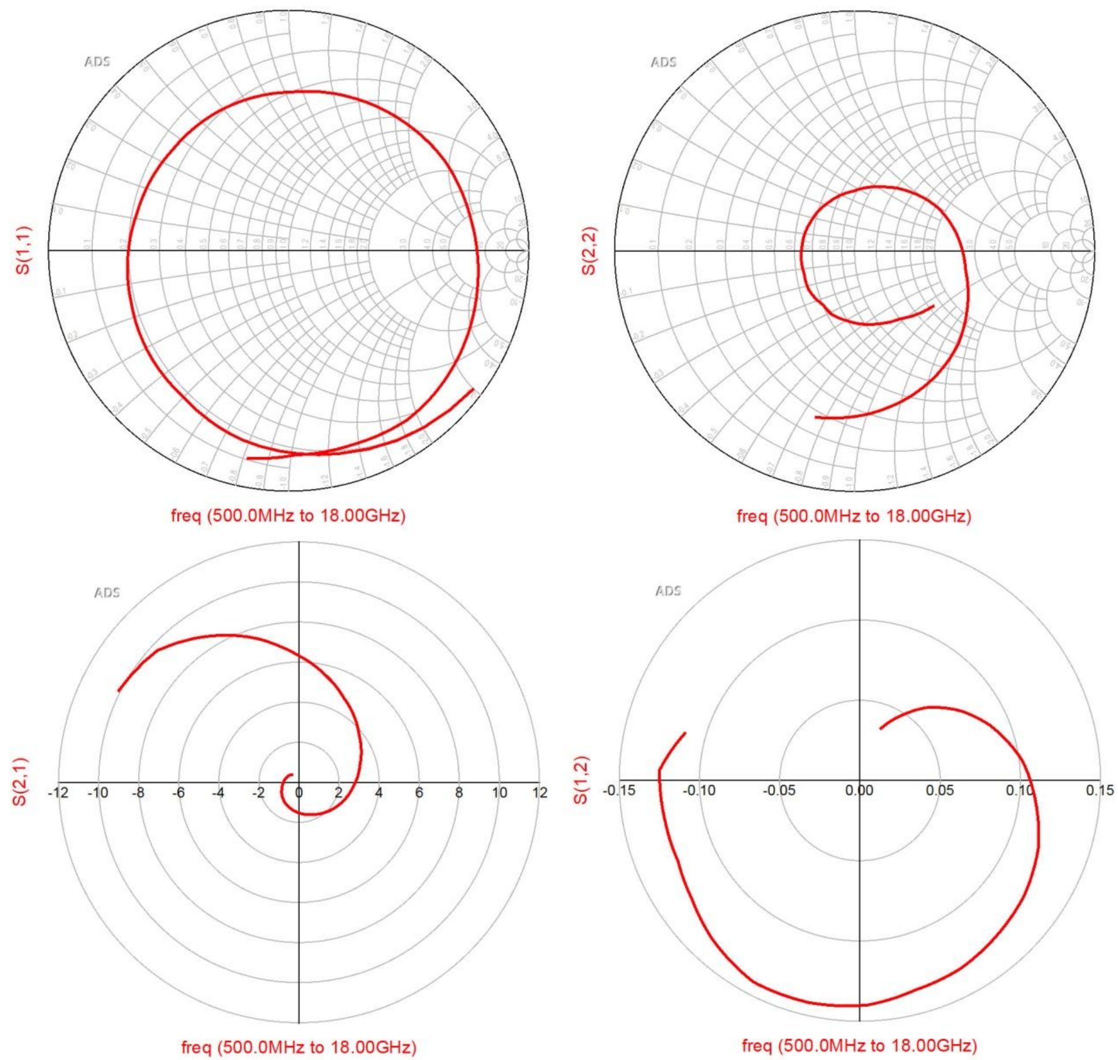
- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz
  - $S11 = 0.64\angle 139^\circ$
  - $S12 = 0.119\angle -21^\circ$
  - $S21 = 3.165\angle 16^\circ$
  - $S22 = 0.22\angle 146^\circ$

```
!ATF-34143
!S-PARAMETERS at Vds=3V  Id=20mA.   LAST UPDATED 01-29-99

# ghz s ma r 50

2.0  0.75  -126  6.306  90  0.088  23  0.26  -120
2.5  0.72  -145  5.438  75  0.095  15  0.25  -140
3.0  0.69  -162  4.762  62  0.102  7  0.23  -156
4.0  0.65  166  3.806  38  0.111  -8  0.22  174
5.0  0.64  139  3.165  16  0.119  -21  0.22  146
6.0  0.65  114  2.706  -5  0.125  -35  0.23  118
7.0  0.66  89  2.326  -27  0.129  -49  0.25  91
8.0  0.69  67  2.017  -47  0.133  -62  0.29  67
9.0  0.72  48  1.758  -66  0.135  -75  0.34  46

!FREQ   Fopt   GAMMA OPT     RN/Zo
!GHZ    dB    MAG    ANG     -

2.0  0.19  0.71  66  0.09
2.5  0.23  0.65  83  0.07
3.0  0.29  0.59  102  0.06
4.0  0.42  0.51  138  0.03
5.0  0.54  0.45  174  0.03
6.0  0.67  0.42  -151  0.05
7.0  0.79  0.42  -118  0.10
8.0  0.92  0.45  -88  0.18
9.0  1.04  0.51  -63  0.30
10.0  1.16  0.61  -43  0.46
```

$$
\begin{cases}
S_{11} = 0.64\angle 139^\circ \\
S_{11} = 0.64\cdot\cos 139^\circ + j\cdot 0.64\cdot\sin 139^\circ \\
S_{11} = -0.4830 + j\cdot 0.4199
\end{cases}
$$

# Example

- ATF-34143
- **at**
  - **Vds=3V**
  - **Id=20mA.**



# Solution + region identification

- S parameters
  - S11 = -0.483+0.42·j
  - S12 = 0.111-0.043·j
  - S21 = 3.042+0.872·j
  - S22 = -0.182+0.123·j
- | S11 | = 0.64 < 1
- $|C_L| < R_L$ , 0∈CSOUT
- The center of the Smith chart is placed inside the output stability circle (0∈CSOUT) and is a stable point (| S11 | < 1)
  - the inside of the output stability circle – stability region
  - the outside of the output stability circle – instability region

$$C_L = \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} = 3.931 - 0.897 \cdot j$$

$$|C_L| = 4.032$$

$$R_L = \frac{|S_{12} \cdot S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|} = 4.891$$

# Solution + region identification

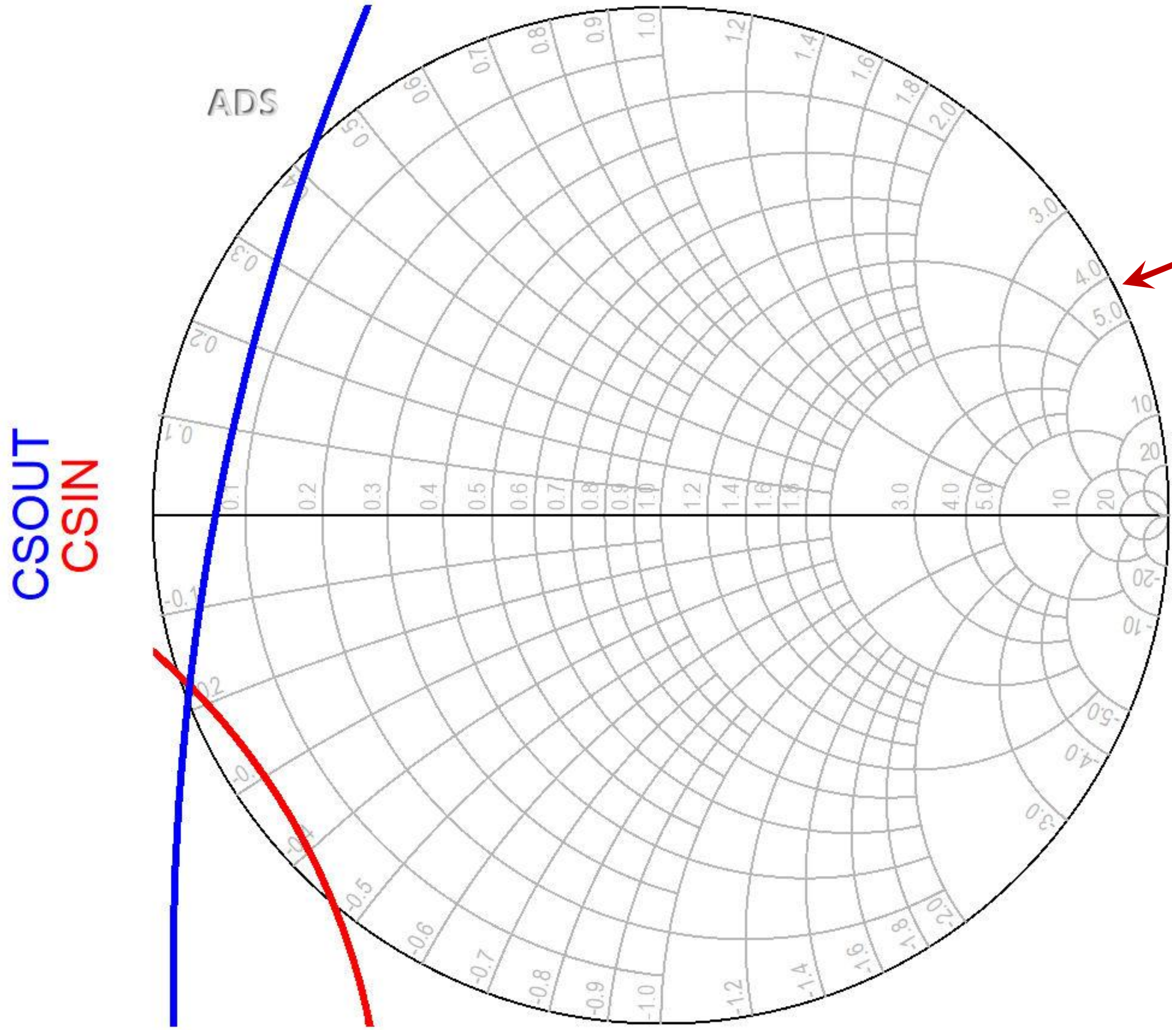
- S parameters
  - S11 = -0.483+0.42·j
  - S12 = 0.111-0.043·j
  - S21 = 3.042+0.872·j
  - S22 = -0.182+0.123·j
- | S22 | = 0.22 < 1
- $|C_S| > R_S$ , $0 \notin CSIN$
- The center of the Smith chart is placed outside the input stability circle ($0 \notin CSIN$) and is a stable point (| S22 | < 1)
  - the outside of the input stability circle – stability region
  - the inside of the input stability circle – instability region

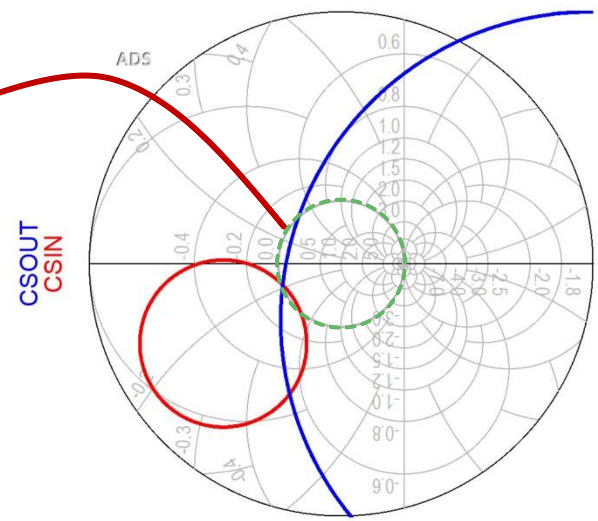
$$C_S = \frac{\left(S_{11} - \Delta \cdot S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} = -1.871 - 1.265 \cdot j$$

$$|C_S| = 2.259$$

$$R_S = \frac{|S_{12} \cdot S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|} = 1.325$$

# ADS



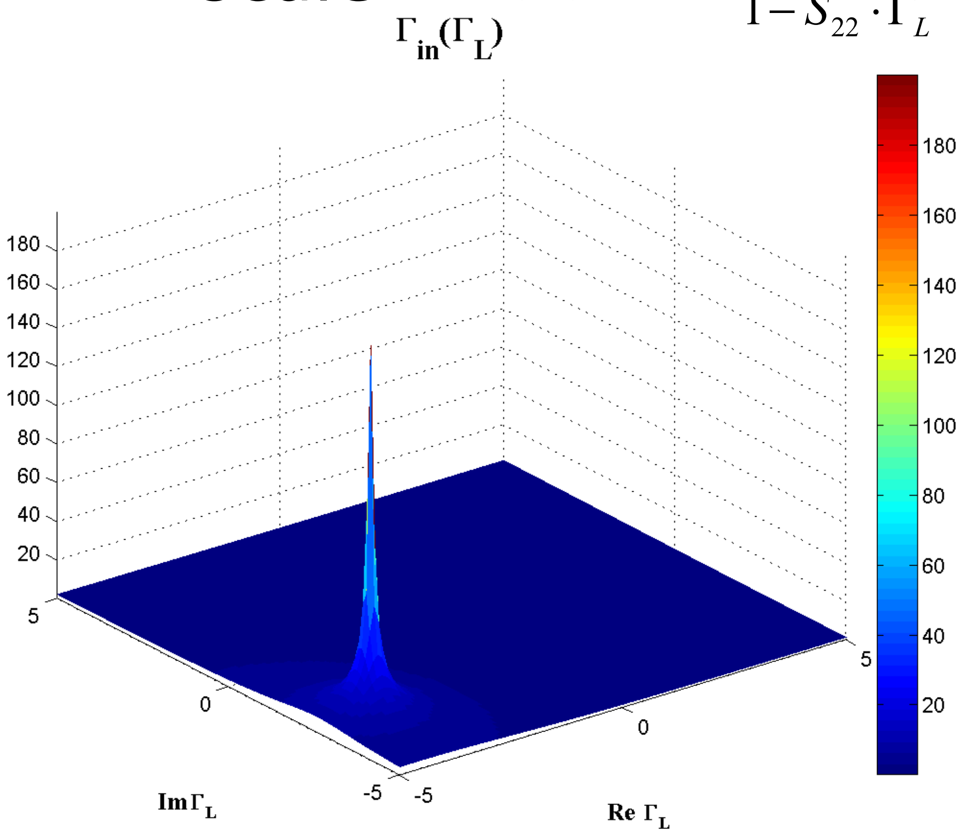



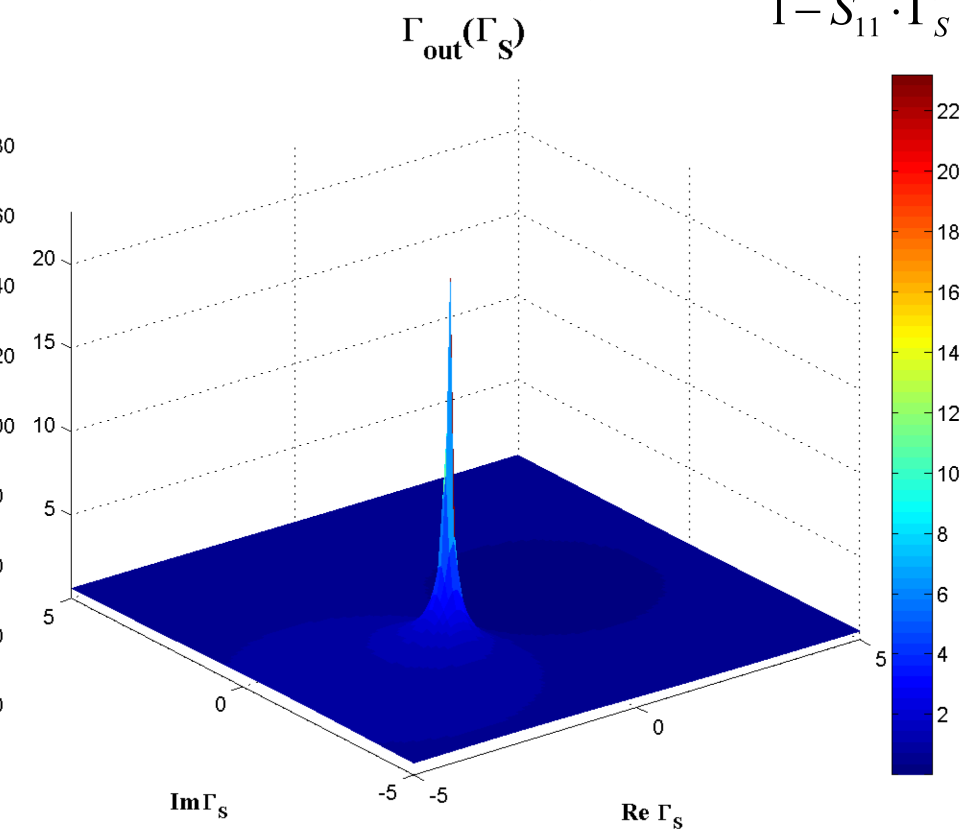
# 3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

- High variations -> we change to z logarithmic scale

$$\Gamma_{in}(\Gamma_L) = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

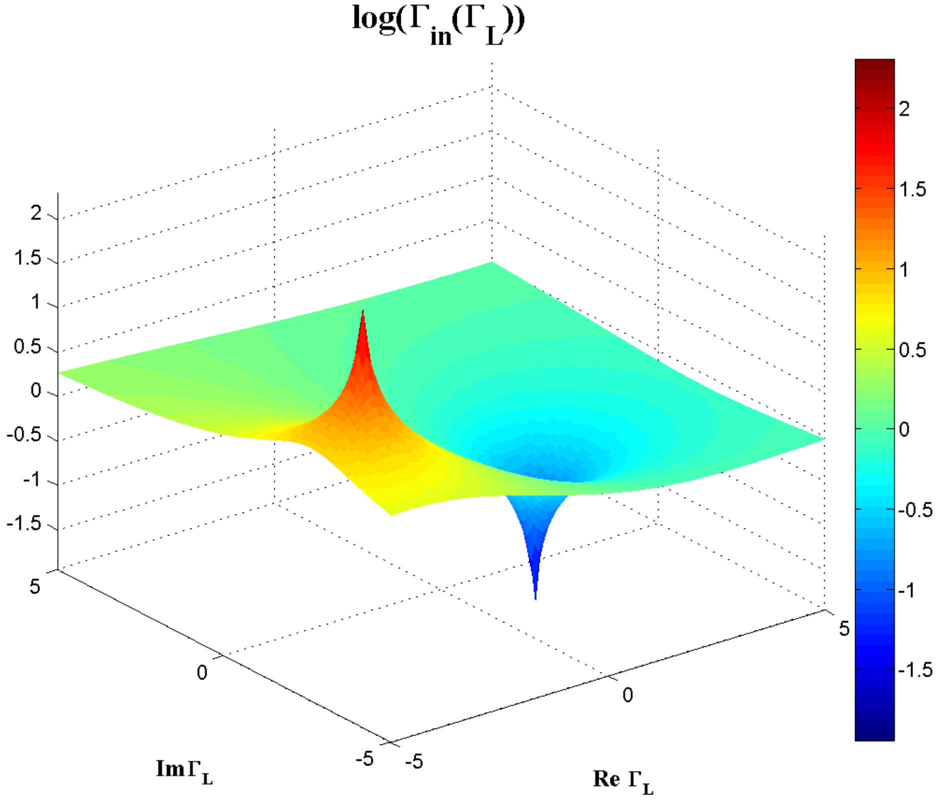
$$\Gamma_{out}(\Gamma_S) = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

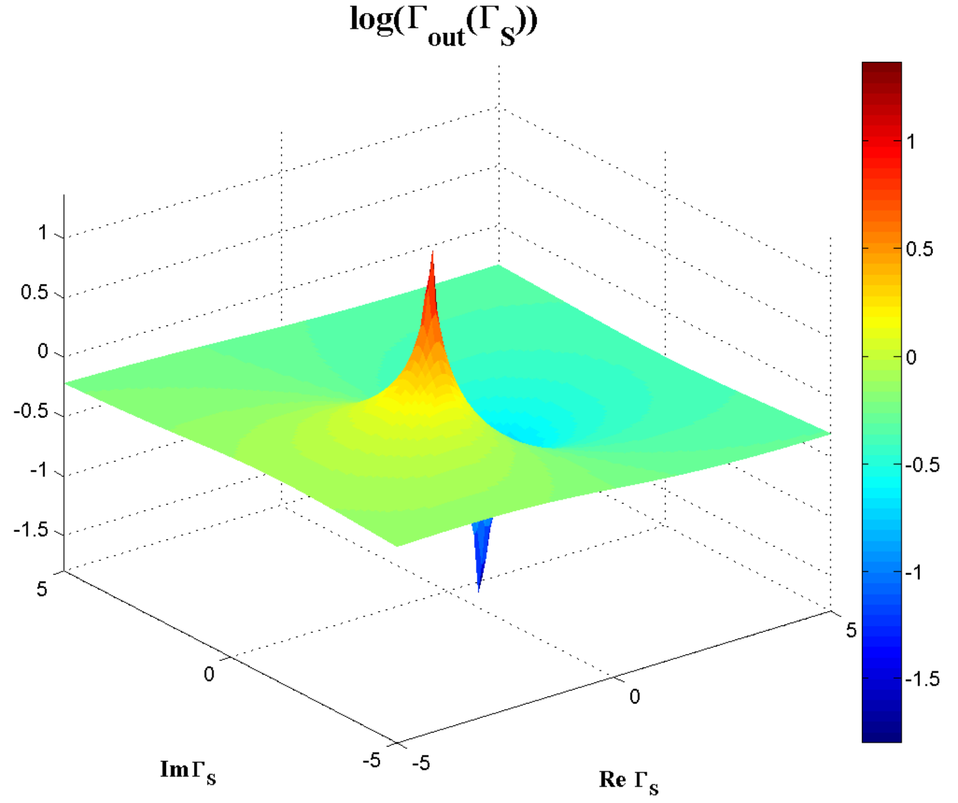


# 3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

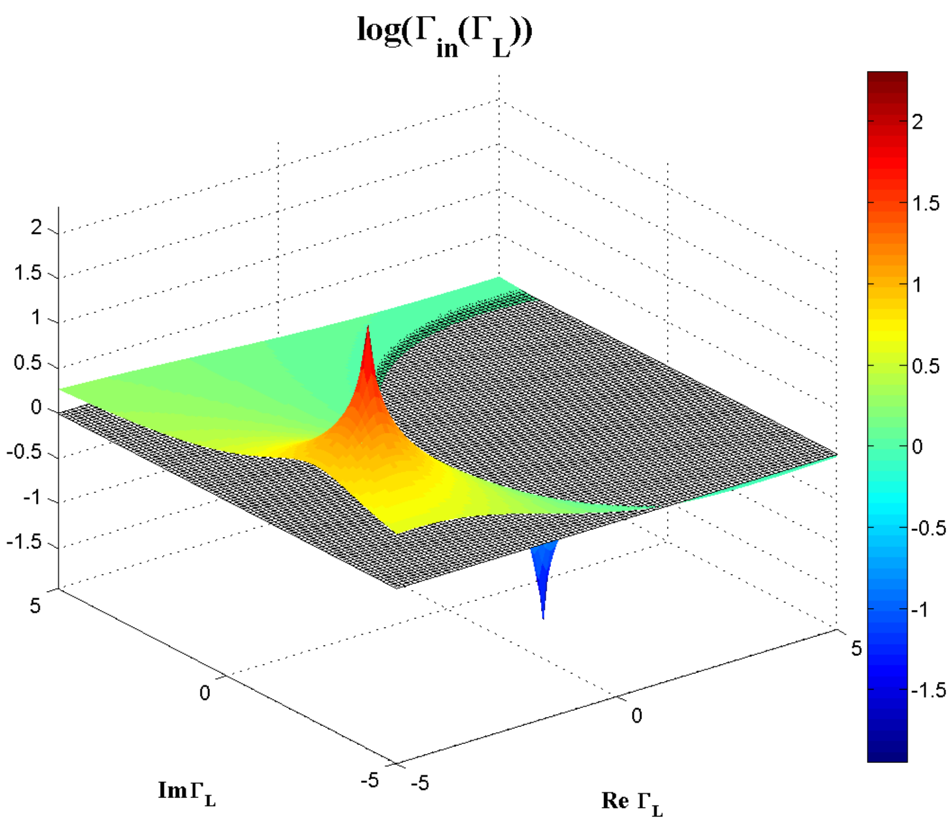
- $\log_{10}|\Gamma_{in}|$, $\log_{10}|\Gamma_{out}|$

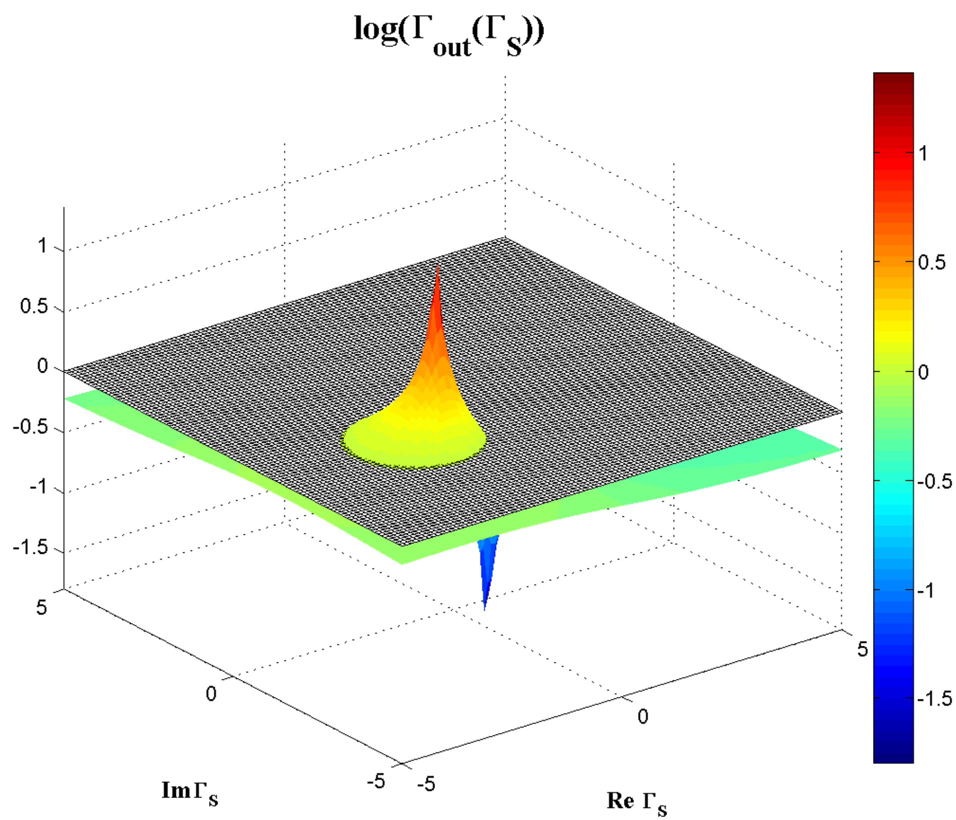


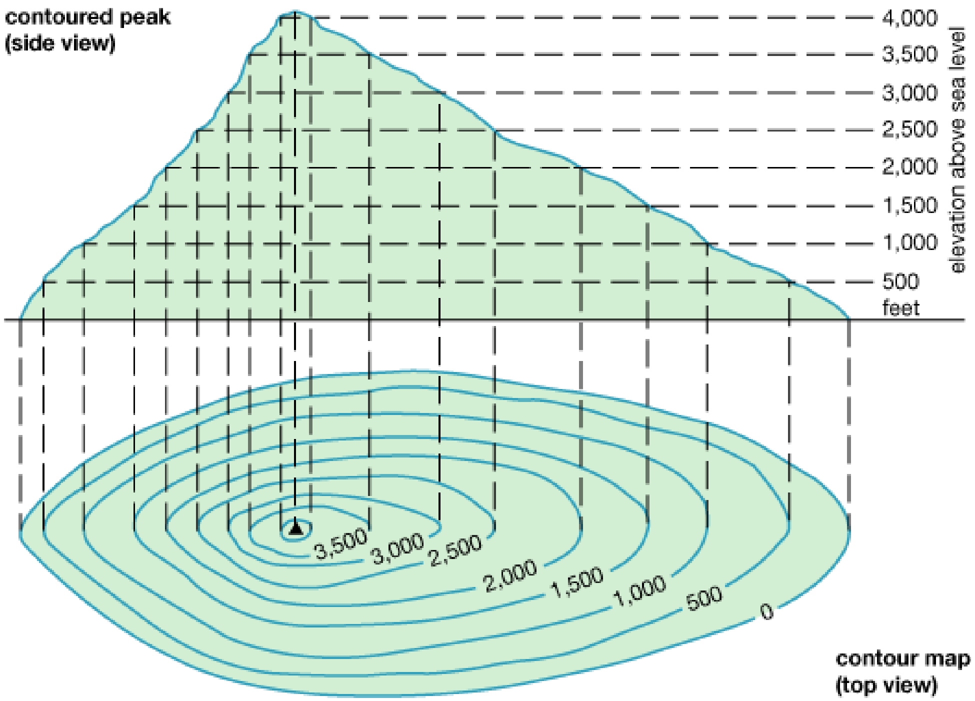
# 3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$, $|\Gamma|=1$

- $|\Gamma| = 1 \rightarrow \log_{10}|\Gamma| = 0$, the intersection with the plane z = 0 is a circle

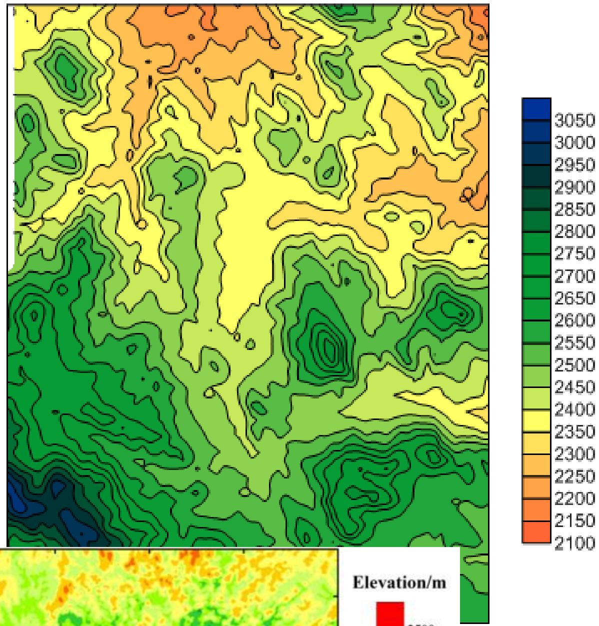


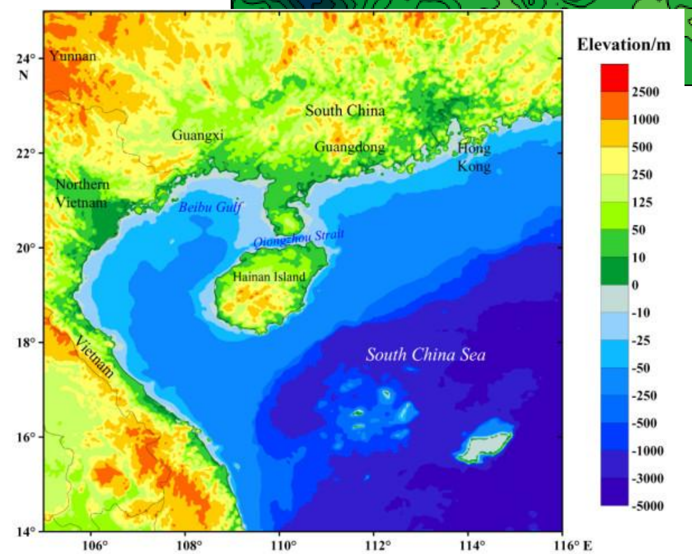



# Contour map/lines





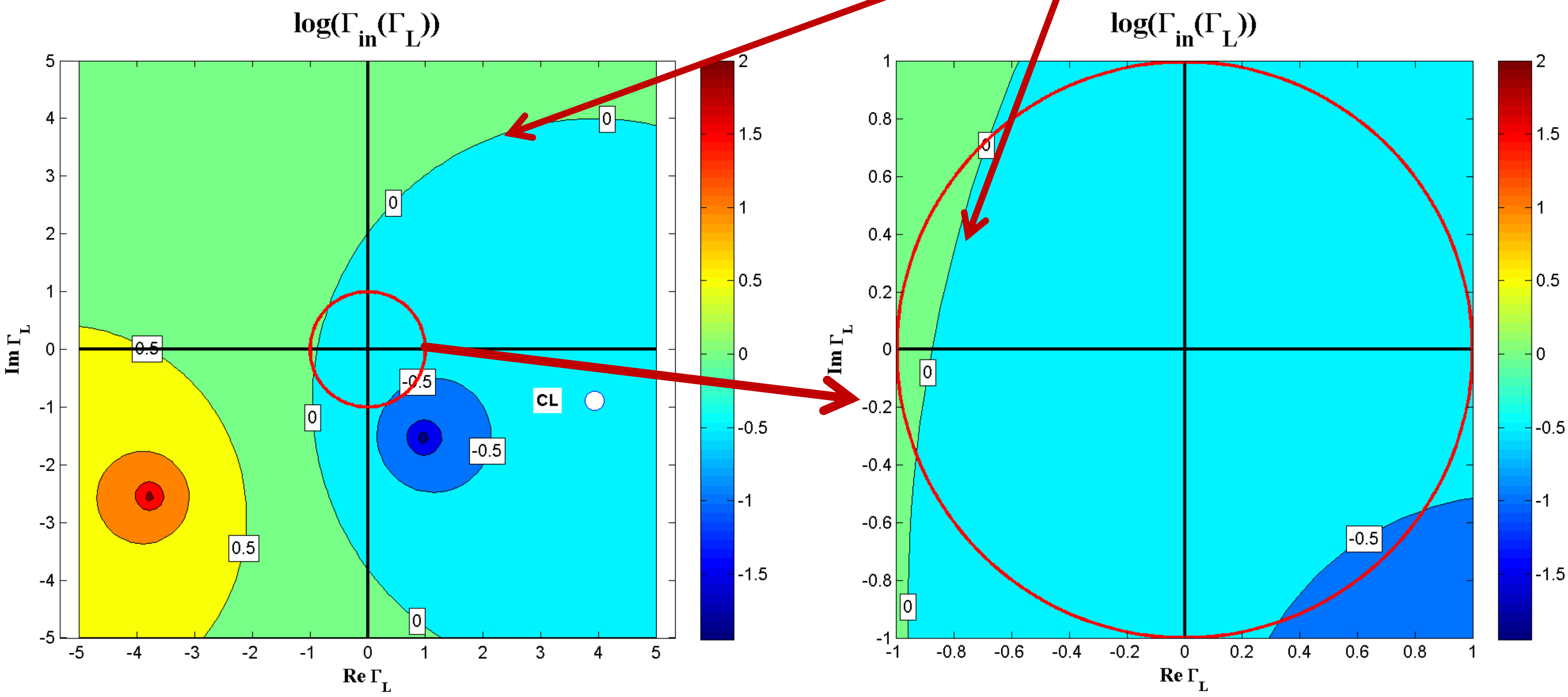



# Contour lines of $\log_{10}|\Gamma_{in}|$

- $\log_{10}|\Gamma_{in}| = 0$, $\Gamma_L$, CSOUT

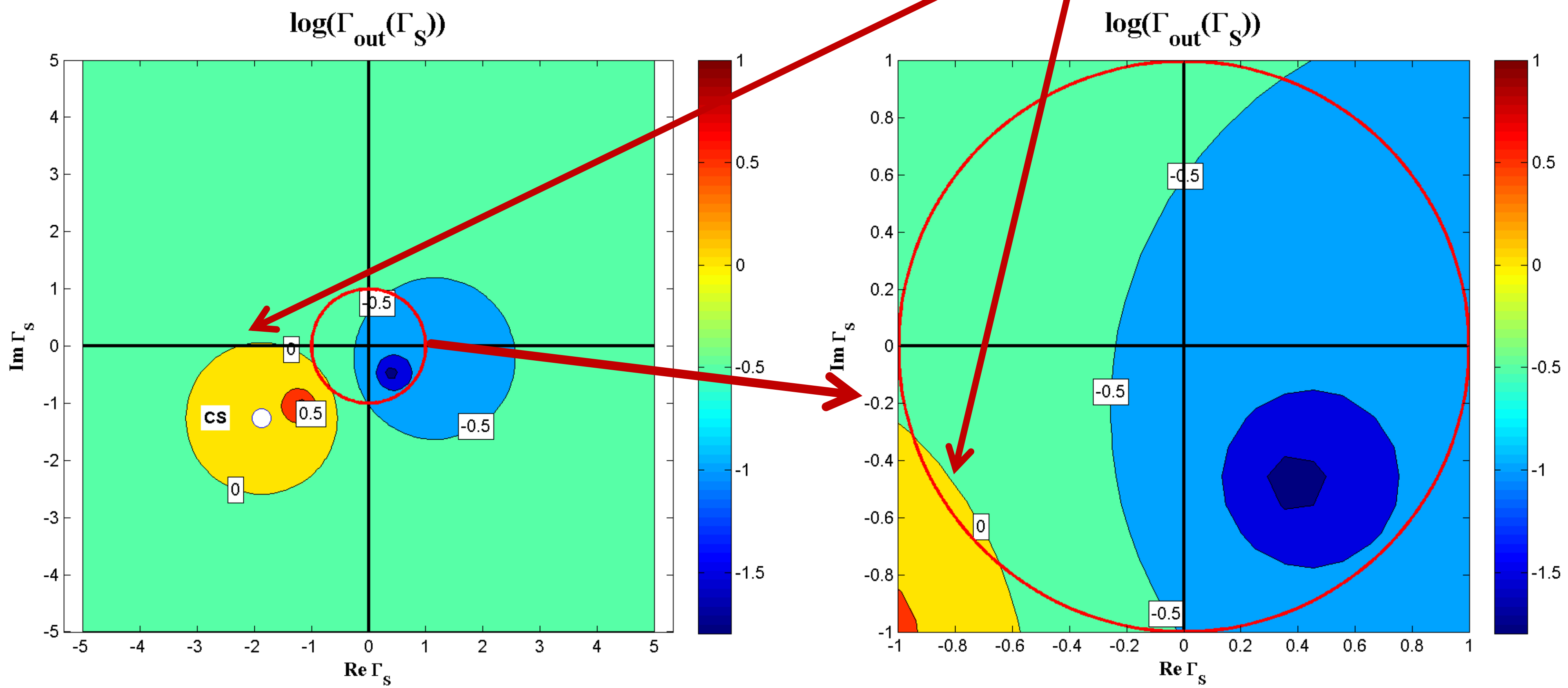
$\log_{10}|\Gamma_{in}| = 0$,
$|\Gamma_{in}| = 1$



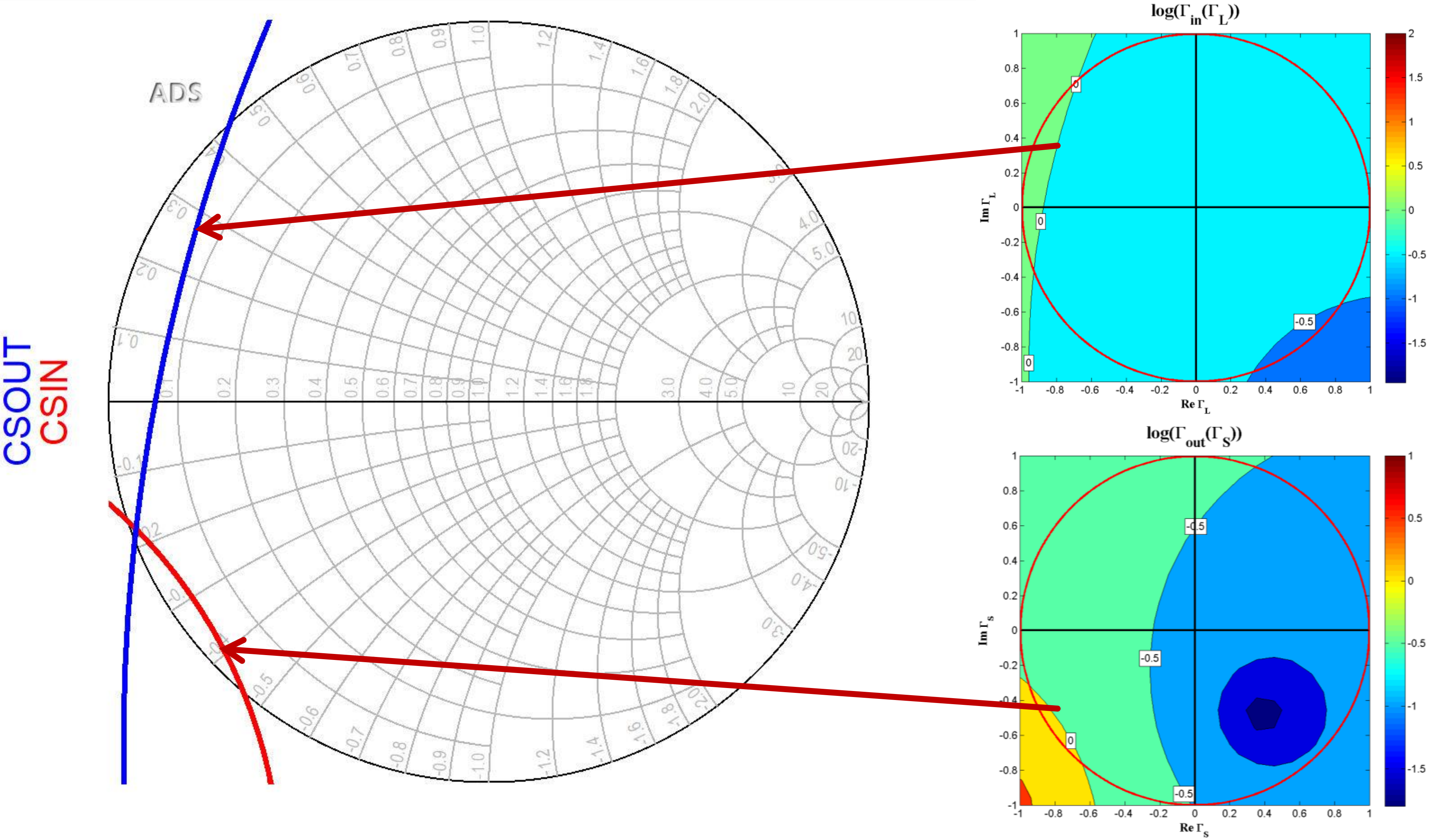
# Contour lines of $\log_{10}|\Gamma_{out}|$

- $\log_{10}|\Gamma_{out}| = 0$, $\Gamma_S$, CSIN

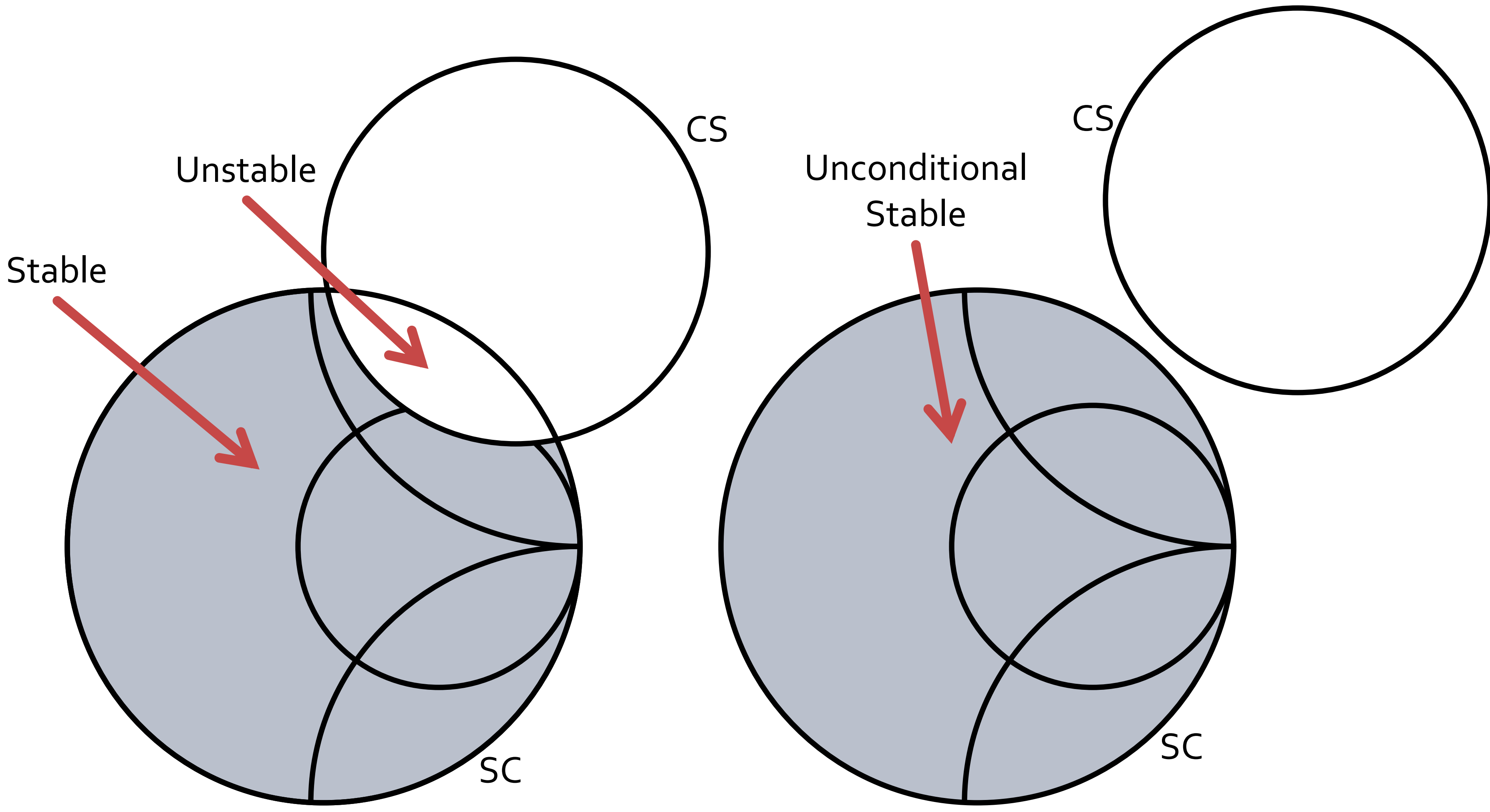
$\log_{10}|\Gamma_{out}| = 0$,
$|\Gamma_{out}| = 1$



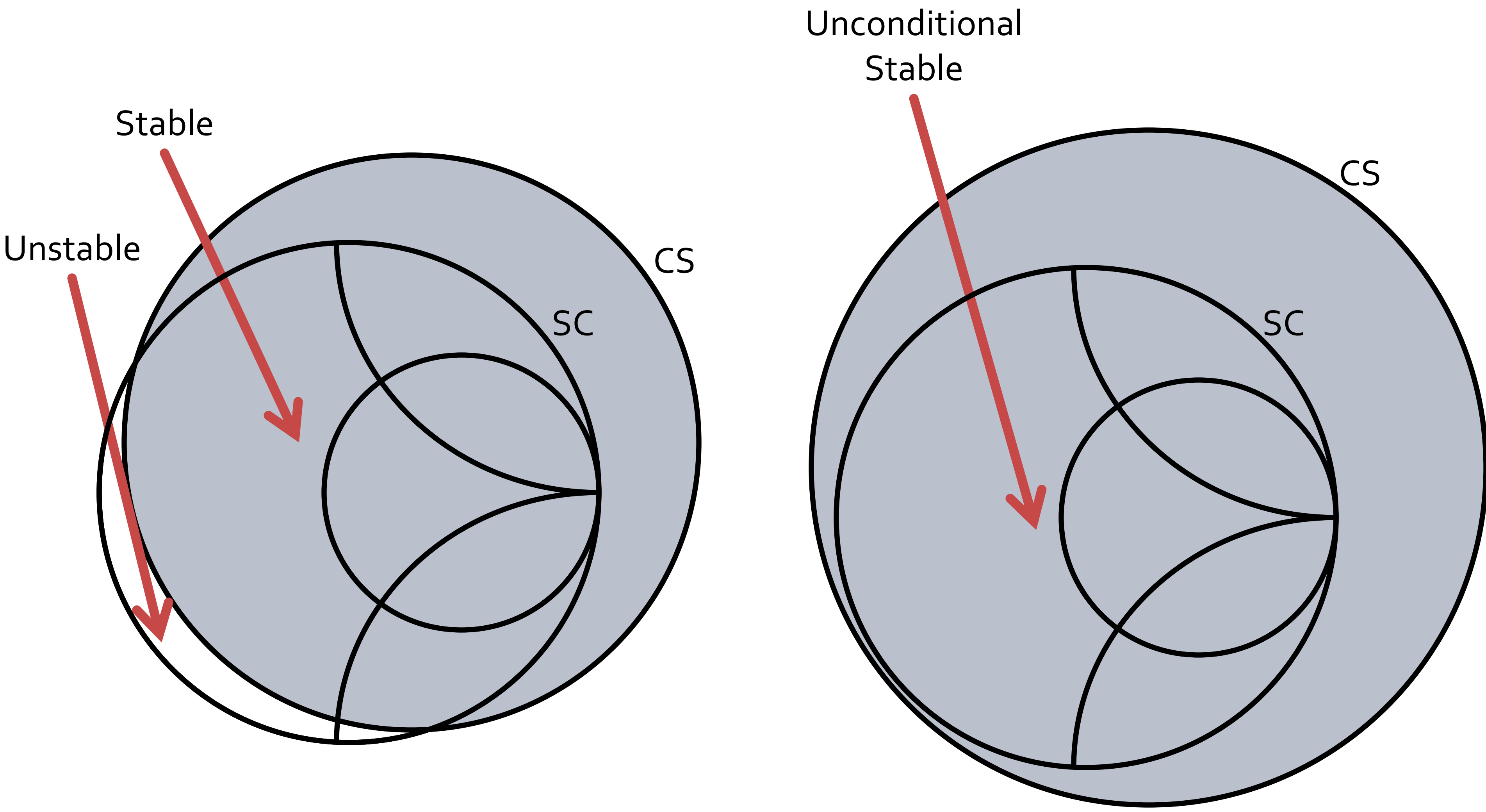
# CSIN, CSOUT



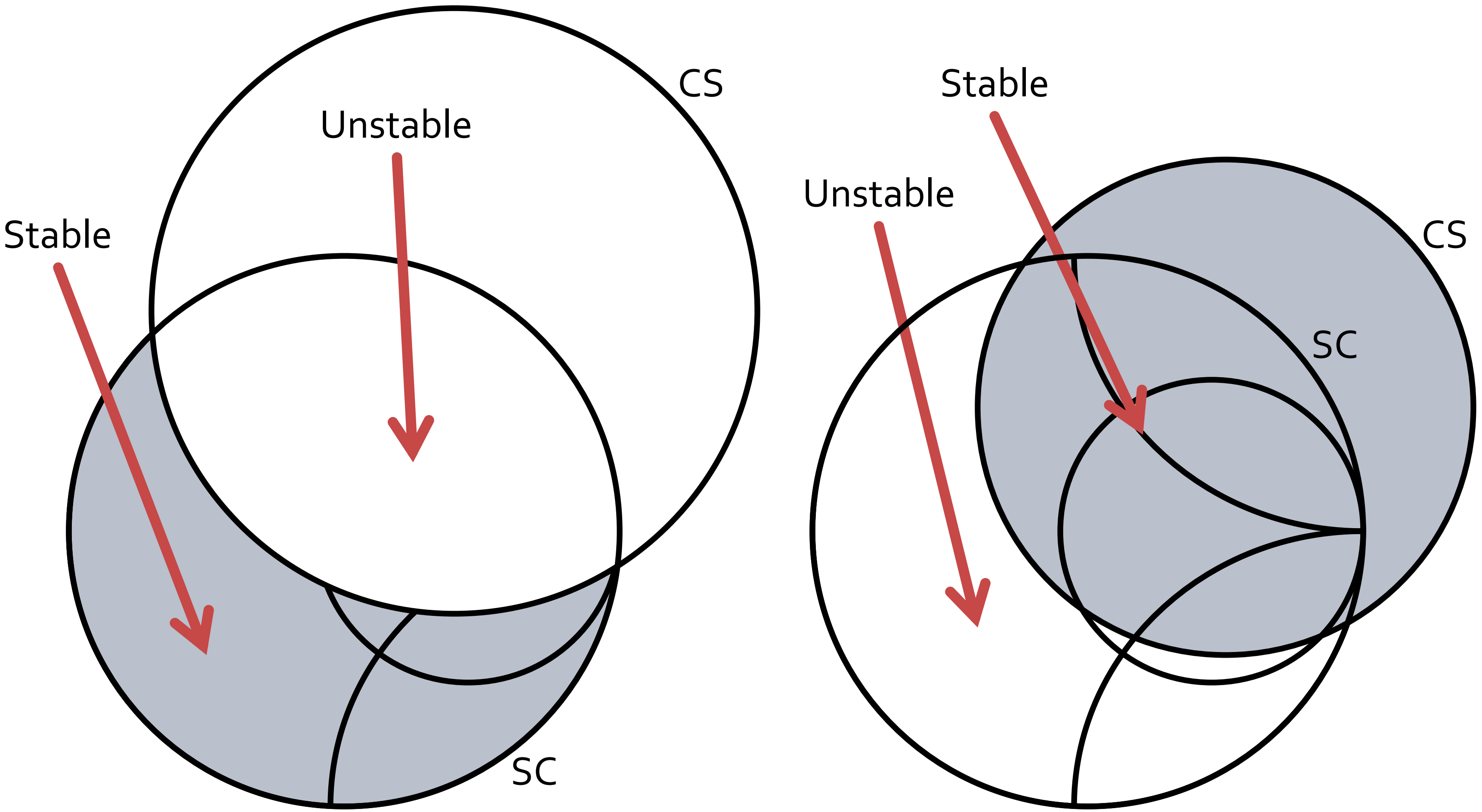
# Several possible positioning



# Several possible positioning



# (Quite) Rare positioning



# Stability

- **Unconditional stability:** the circuit is unconditionally stable if $|\Gamma_{in}|<1$ and $|\Gamma_{out}|<1$ for **any** passive impedance of the load/source
- **Conditional stability:** the circuit is conditionally stable if $|\Gamma_{in}|<1$ and $|\Gamma_{out}|<1$ only for **some** passive impedance of the load/source
  - passive impedance of the load/source <-> interior of the Smith Chart (radius 1 circle in the complex plane)

# Unconditional stability

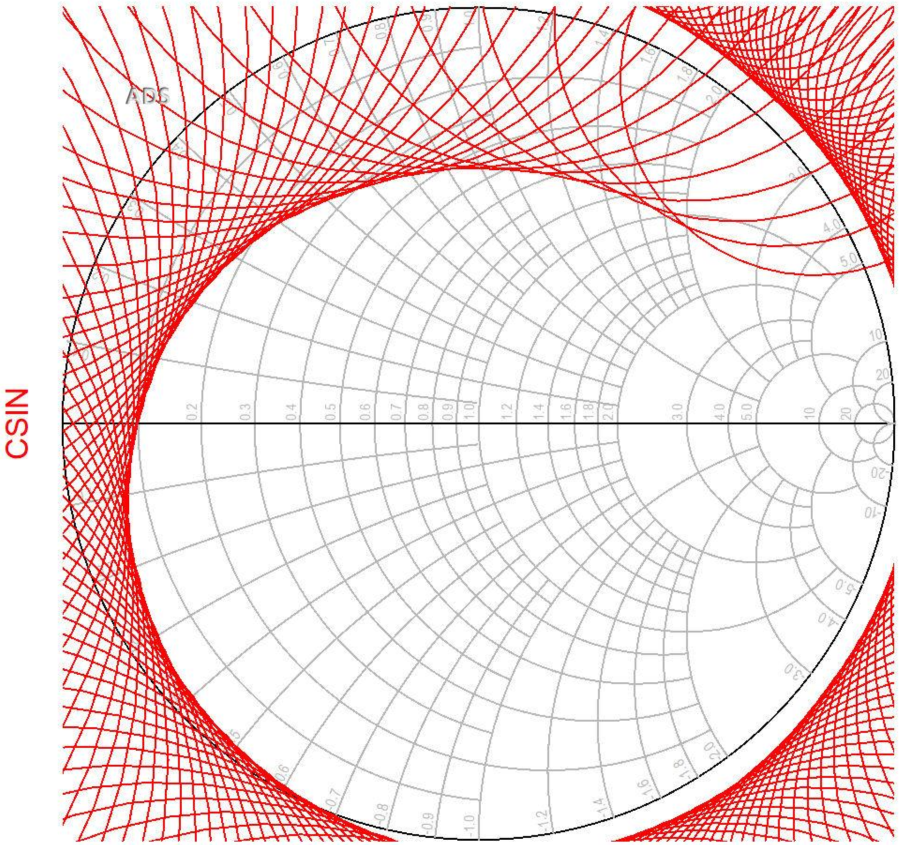
- The two-port is unconditionally stable if either:
  - The stability circle is disjoint with the Smith Chart (exterior to the Chart) and the stable region is outside the circle
  - The stability circle encloses the entire Smith Chart and the stable region is inside the circle
- One mandatory condition for unconditional stability is |S11| < 1 (CSOUT) or |S22| < 1 (CSIN) – if in at least one point the two-port is not stable then it cannot be unconditionally stable
- Mathematically :

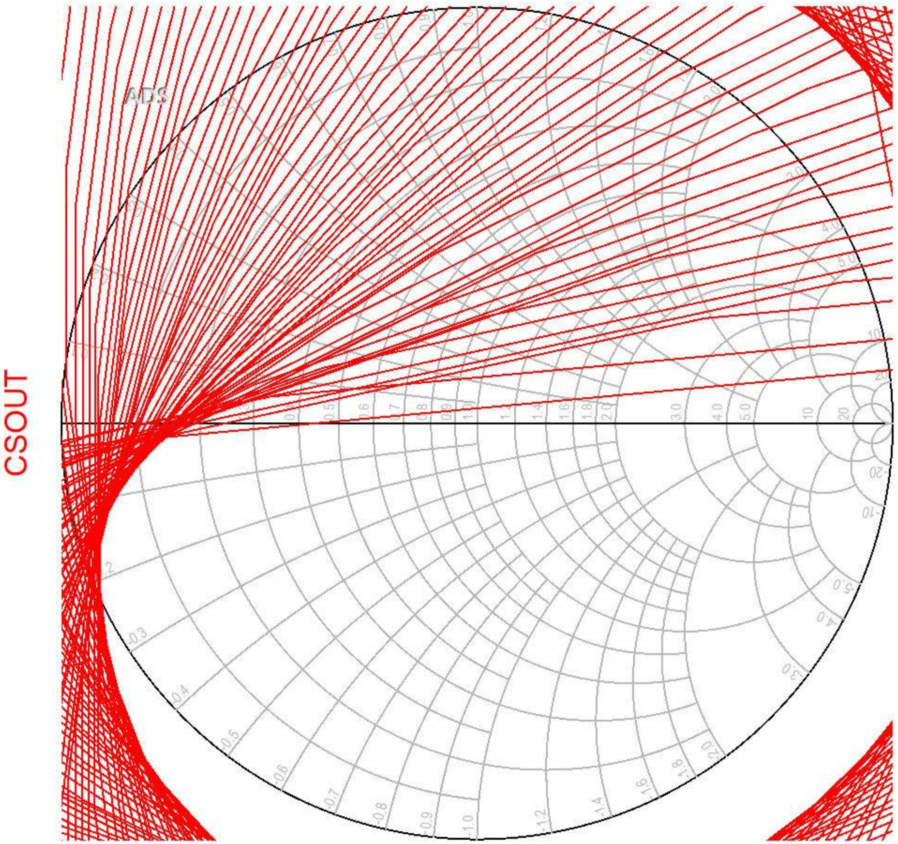
$$\begin{cases} \left| |C_L| - R_L \right| > 1 \\ |S_{11}| < 1 \end{cases} \qquad \begin{cases} \left| |C_S| - R_S \right| > 1 \\ |S_{22}| < 1 \end{cases}$$

# Tests for Unconditional Stability

- Useful for wide frequency range analysis
- It is not enough to check the stability only at the operating frequencies
  - we must obtain stable operation for chosen $\Gamma_L$ and $\Gamma_S$ at **any** frequency

# Circles in wide frequency range



# Rollet's condition

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} \qquad \Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$$

- The two-port is **unconditionally stable** if:
- two conditions are simultaneously satisfied:
  - $K > 1$
  - $|\Delta| < 1$
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} > 1 \qquad |\Delta| = |S_{11} \cdot S_{22} - S_{12} \cdot S_{21}| < 1$$

# μ Criterion

- Rollet's condition cannot be used to compare the relative stability of two or more devices because it involves constraints on two separate parameters, K and Δ

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-\Delta \cdot S_{11}^*| + |S_{12} \cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
  - μ > 1
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$
- In addition, it can be said that larger values of μ imply greater stability
  - μ is the distance from the center of the Smith Chart to the closest output stability circle

# μ' Criterion

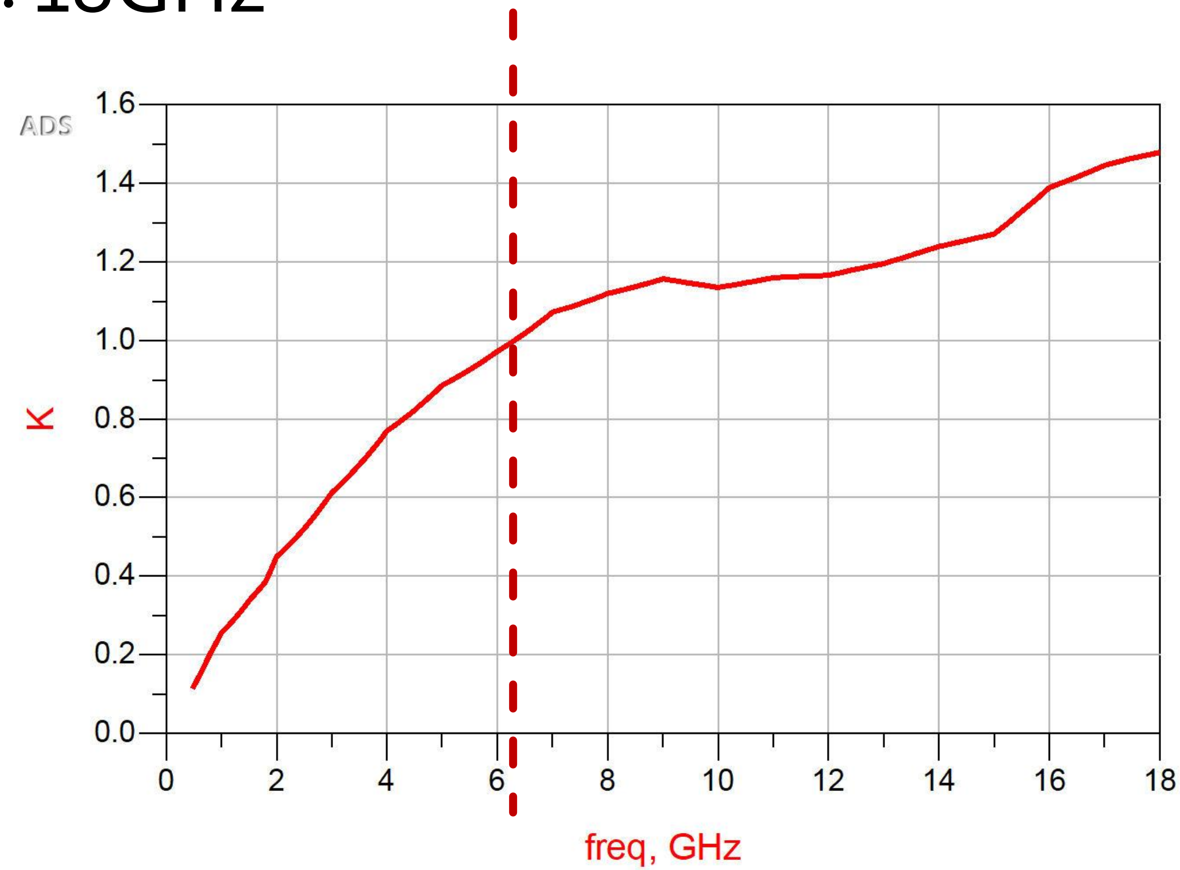
- Dual parameter to μ, determined in relation to the input stability circles

$$\mu' = \frac{1-|S_{22}|^2}{|S_{11}-\Delta\cdot S_{22}^*|+|S_{12}\cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
  - $\mu' > 1$
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$
- In addition, it can be said that larger values of μ' imply greater stability
  - μ' is the distance from the center of the Smith Chart to the closest input stability circle

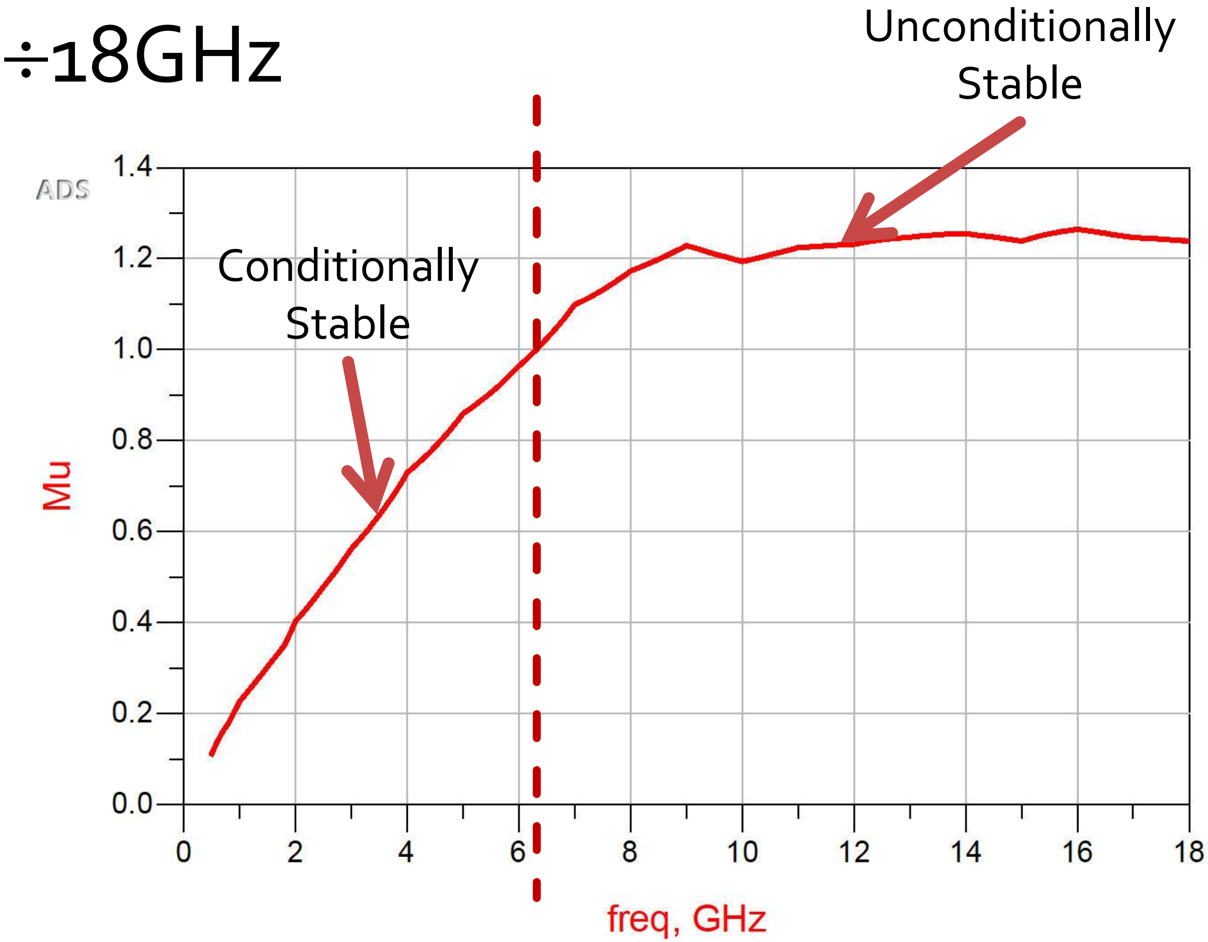
# Rollet's condition

- ATF-34143 **at Vds=3V  Id=20mA**.
- @0.5÷18GHz



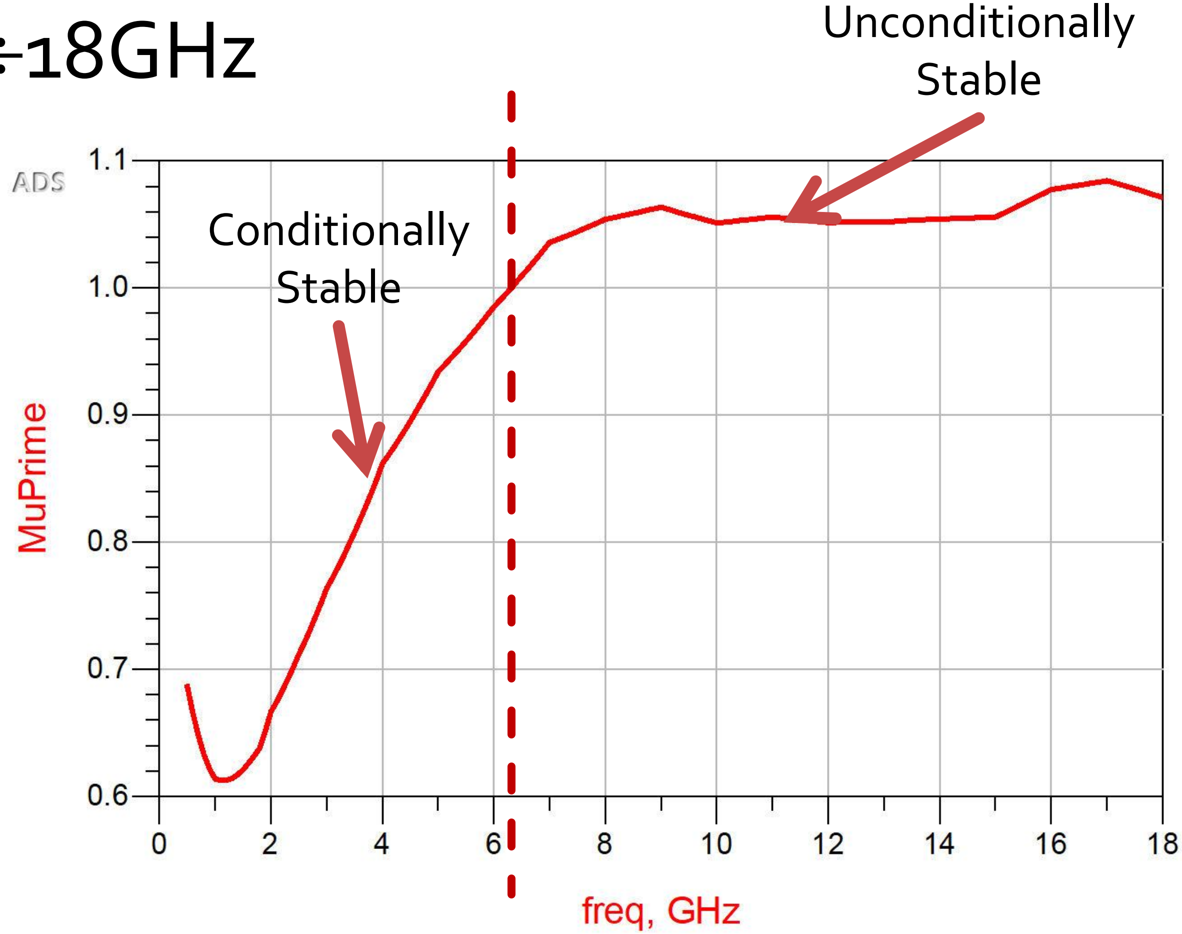
# μ Criterion

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



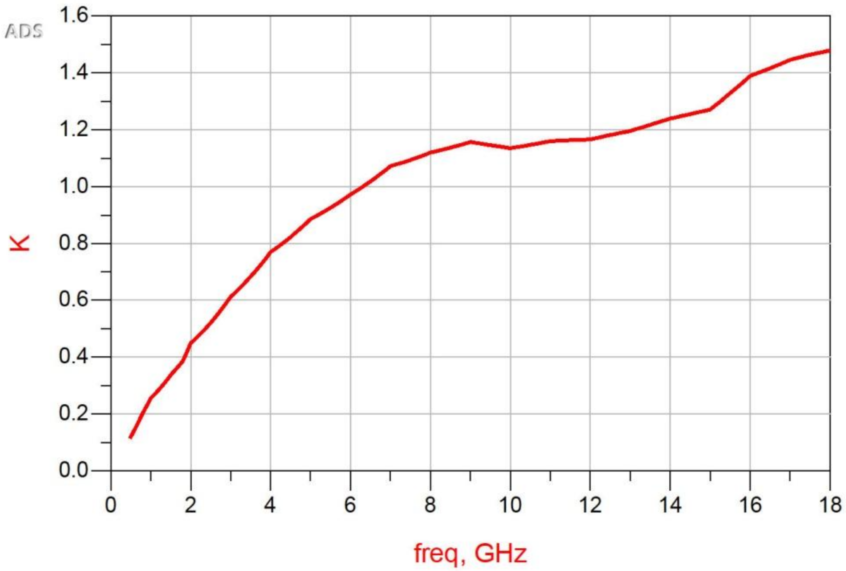
# μ' Criterion

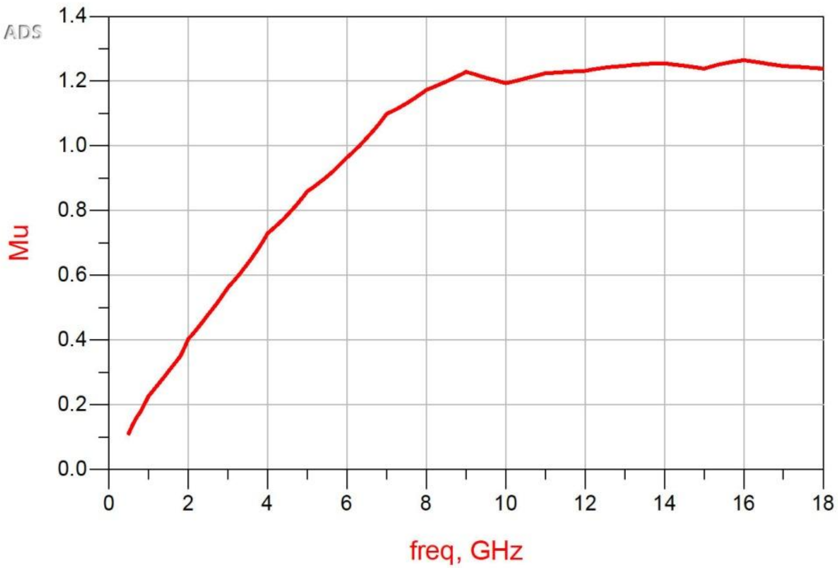
- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



# Stability

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz
- unconditionally stable for f > 6.31GHz

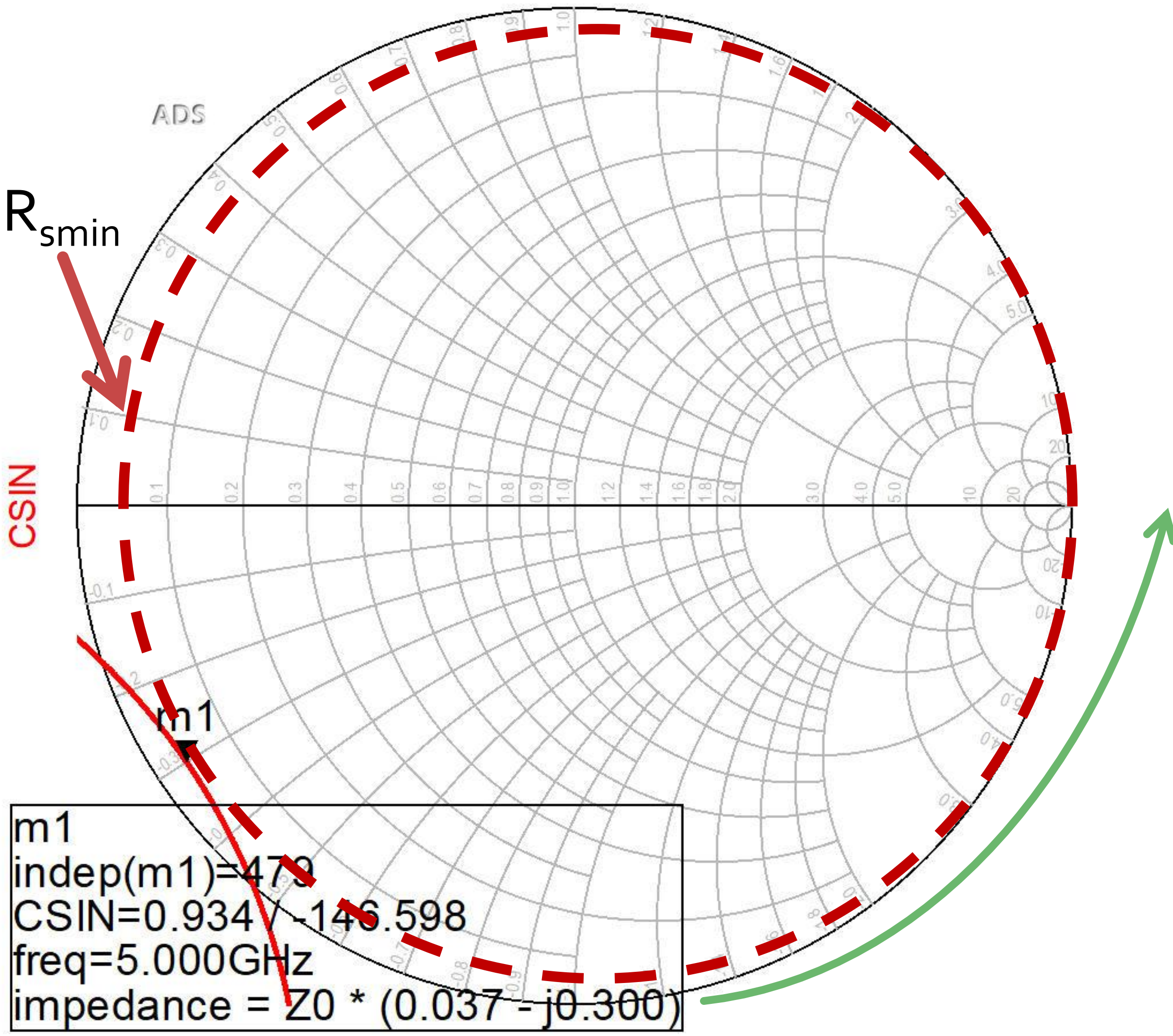


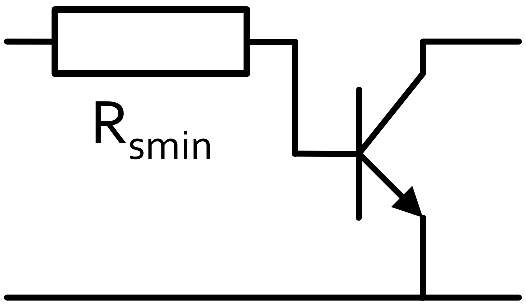



# Stabilization of two-port

- Unconditional stability in a wide frequency range has some important advantages
  - Ex: We can use ATF 34143 to design a (conditionally) stable amplifier at 5GHz, but this design is useless if the amplifier oscillates at 500MHz ($\mu \approx 0.1$)
- **The minimal requirement** when working with conditionally stable devices is to **check stability** at several frequencies over the operating bandwidth and outside the bandwidth
- Unconditional stability can be forced by inserting series/shunt resistors at two-port’s input/output (with loss of gain!)

# Input series resistor

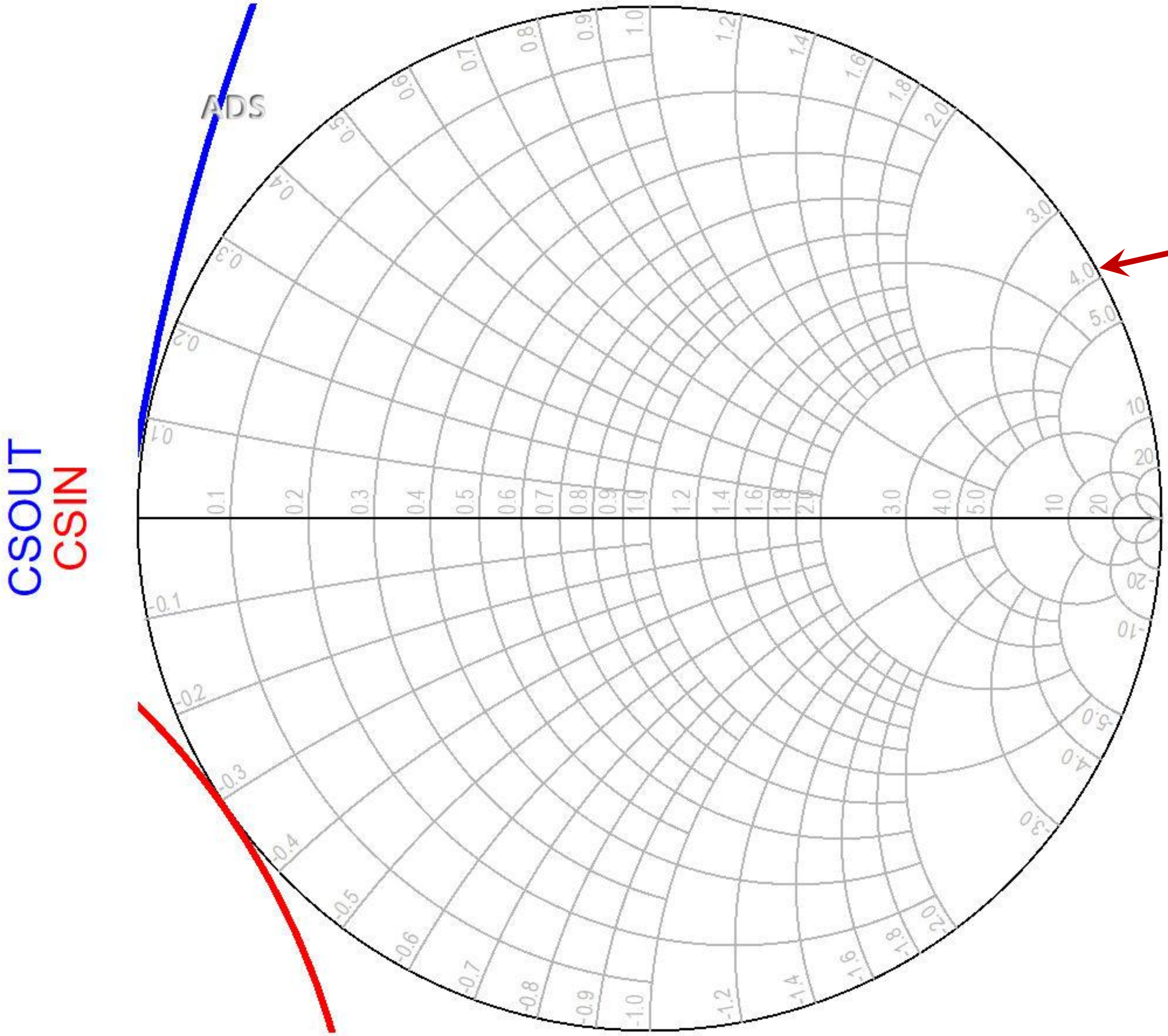


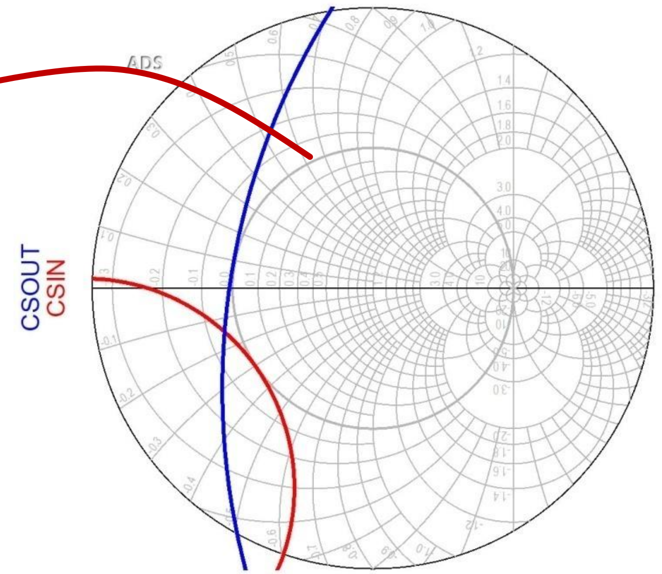



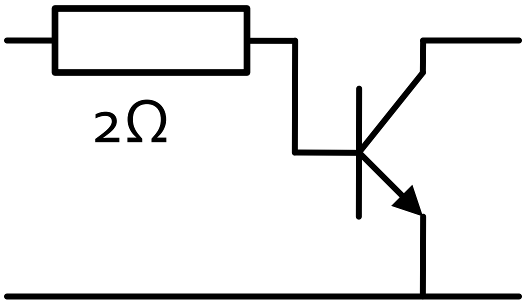

$$z = 0.037 - j \cdot 0.3$$

$$R_{s\,min} = 0.037 \cdot 50\Omega = 1.85\Omega$$

# ADS, Rs = 2Ω

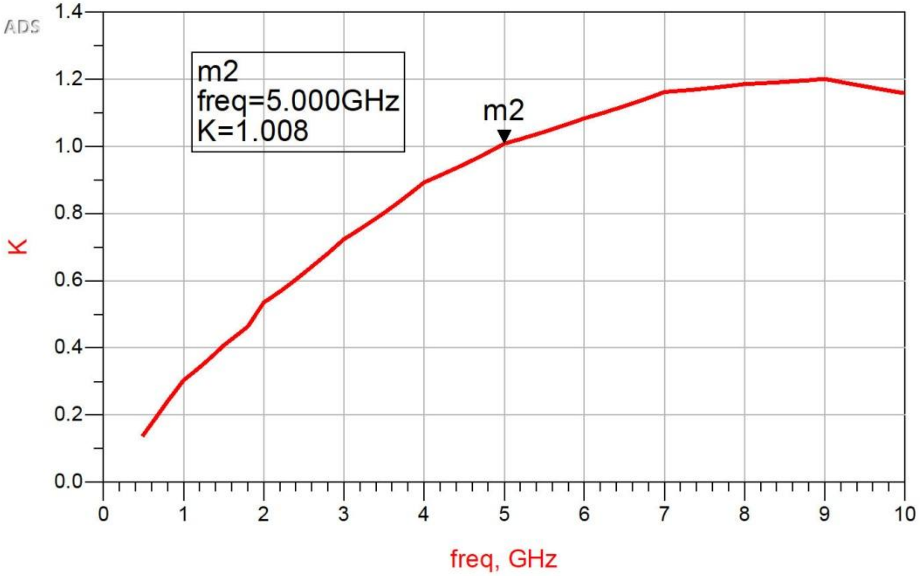


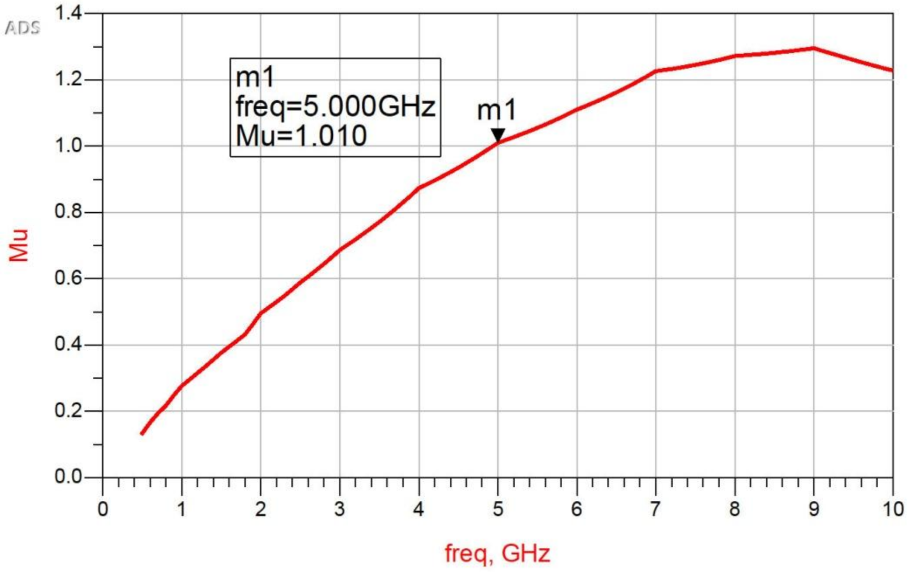



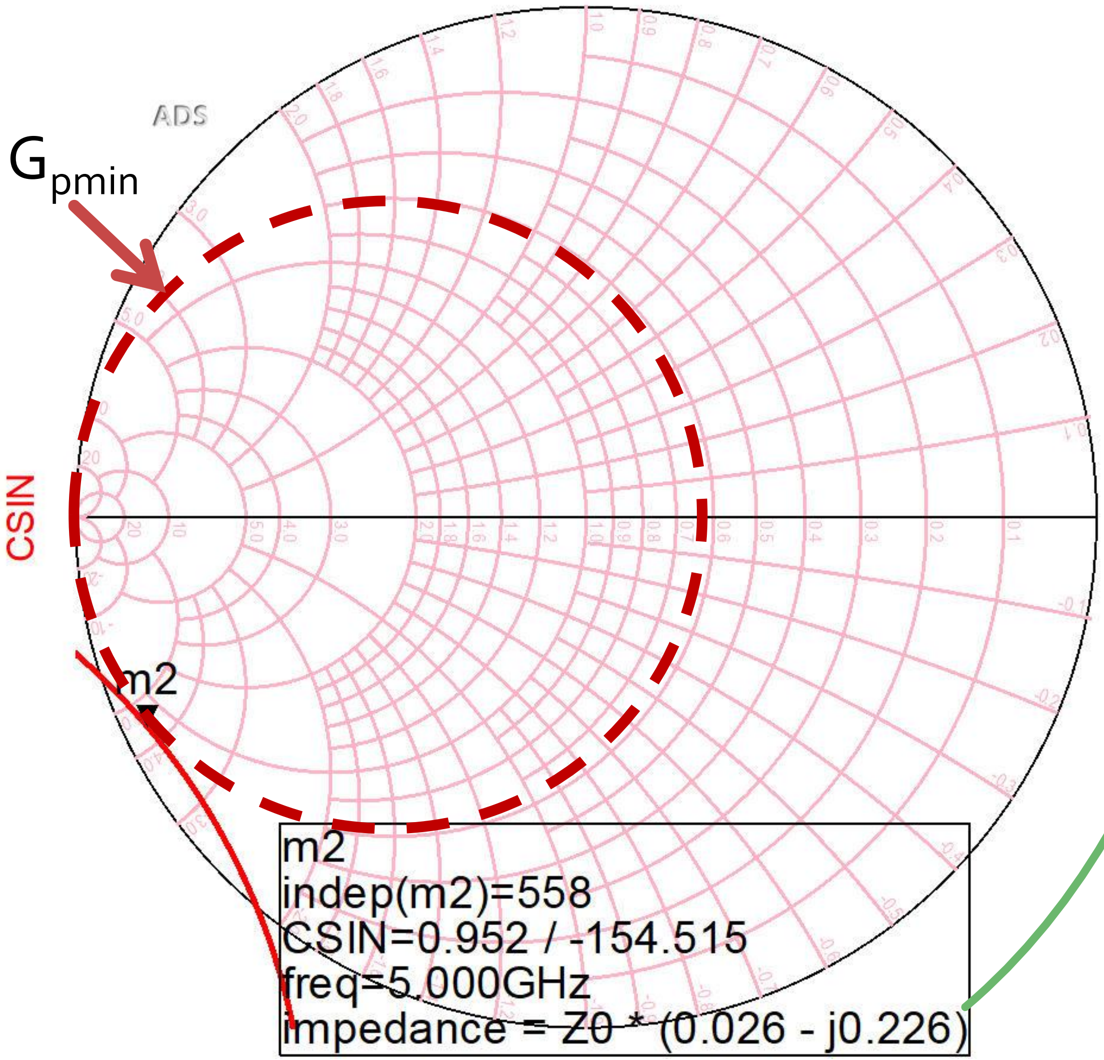
# Input series resistor

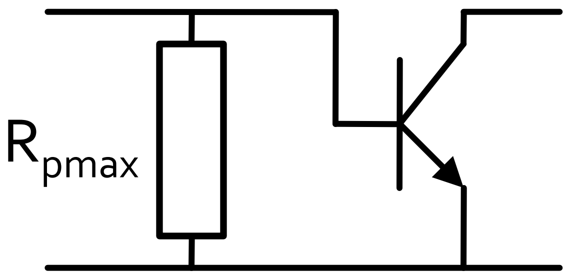
- Rs = 2Ω
- K = 1.008, MAG = 13.694dB @ 5GHz
  - no stabilization, K = 0.886, MAG = 14.248dB @ 5GHz



# Input shunt resistor




$$R_{p\max} = \frac{1}{G_{p\min}}$$

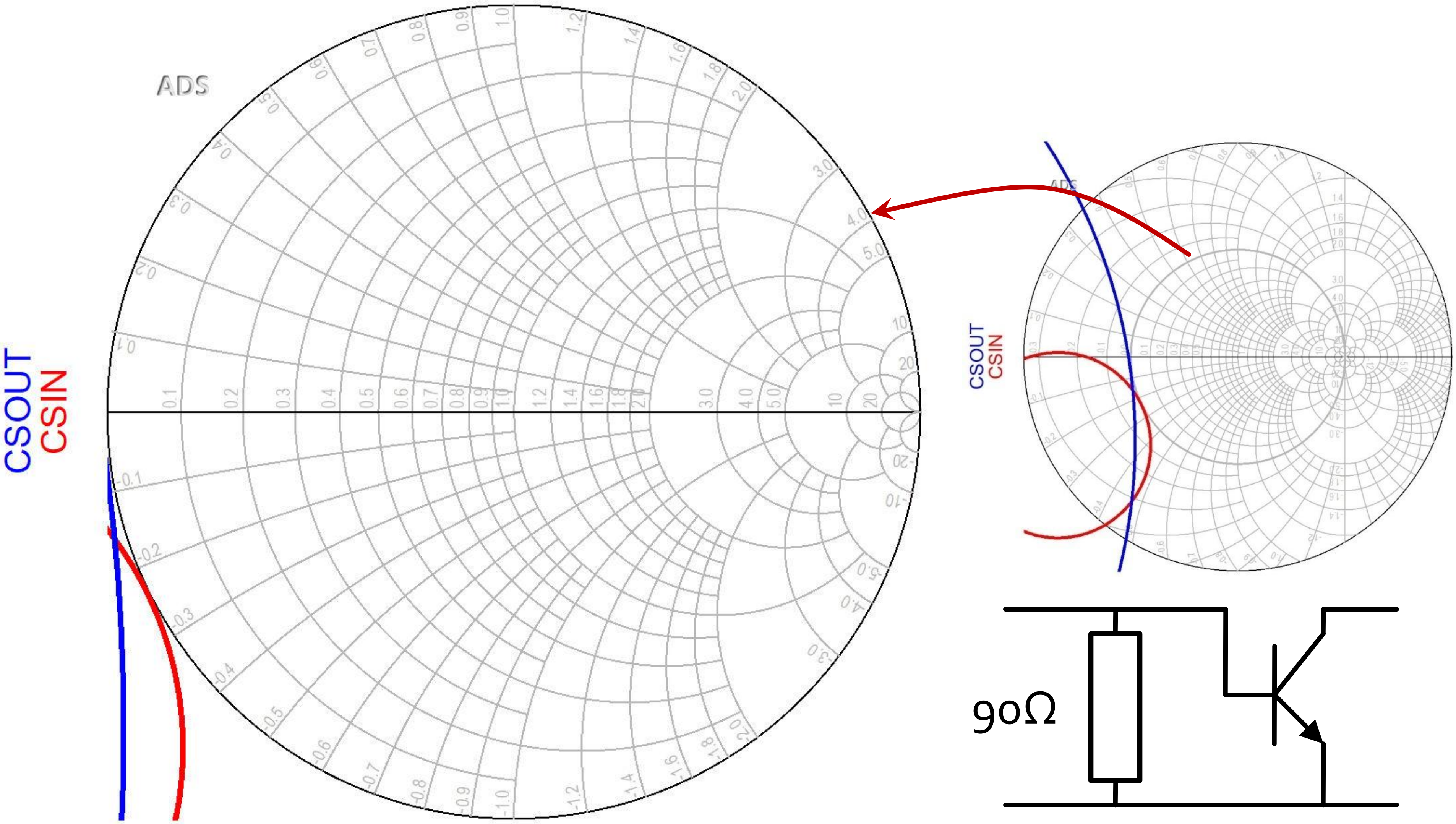
$$z = 0.026 - j \cdot 0.226$$

$$y = \frac{1}{z} = \frac{1}{0.026 - j \cdot 0.226}$$

$$y = 0.502 + j \cdot 4.367$$

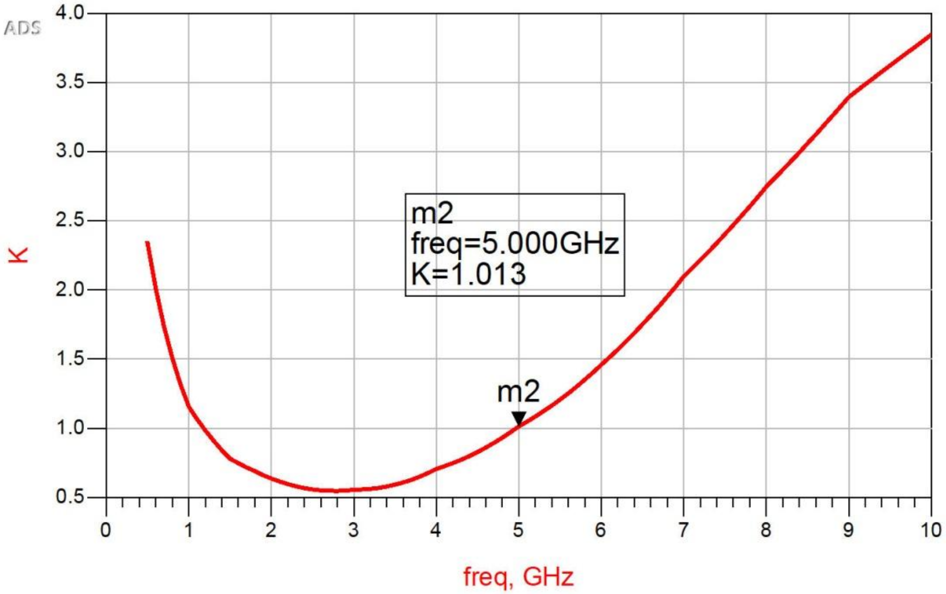
$$R_{p\max} = \frac{50\Omega}{0.502} = 99.6\Omega$$

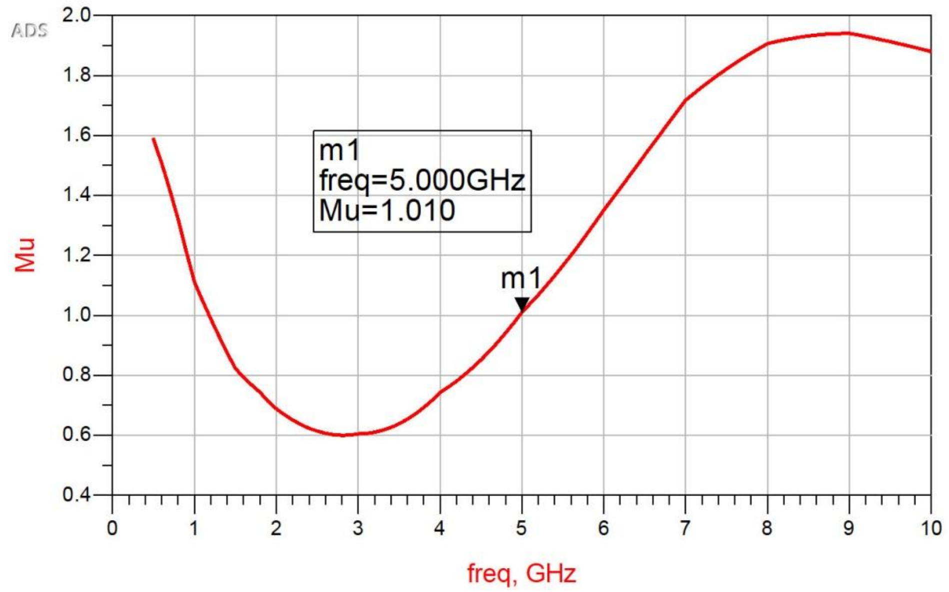
# ADS, Rp = 90Ω



# Input shunt resistor

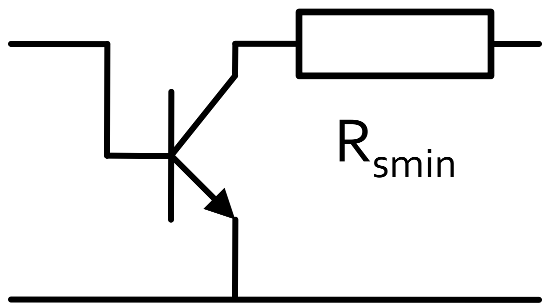
- Rp = 90Ω
- K = 1.013, MAG = 13.561dB @ 5GHz
  - no stabilization, K = 0.886, MAG = 14.248dB @ 5GHz

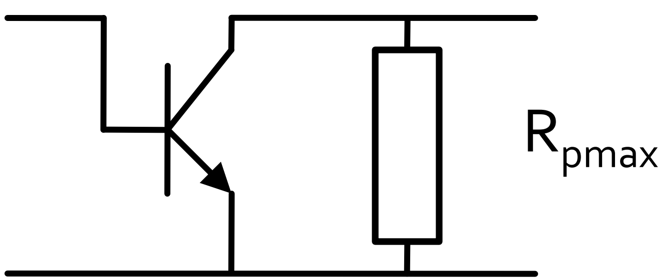


# Output series/shunt resistor

- The procedure can be applied similarly at the output (finding g/r circles tangent to CSOUT)
- From previous examples, resistive loading at the input has a positive effect over output stability and vice versa (resistive loading at the output, effect over input stability)

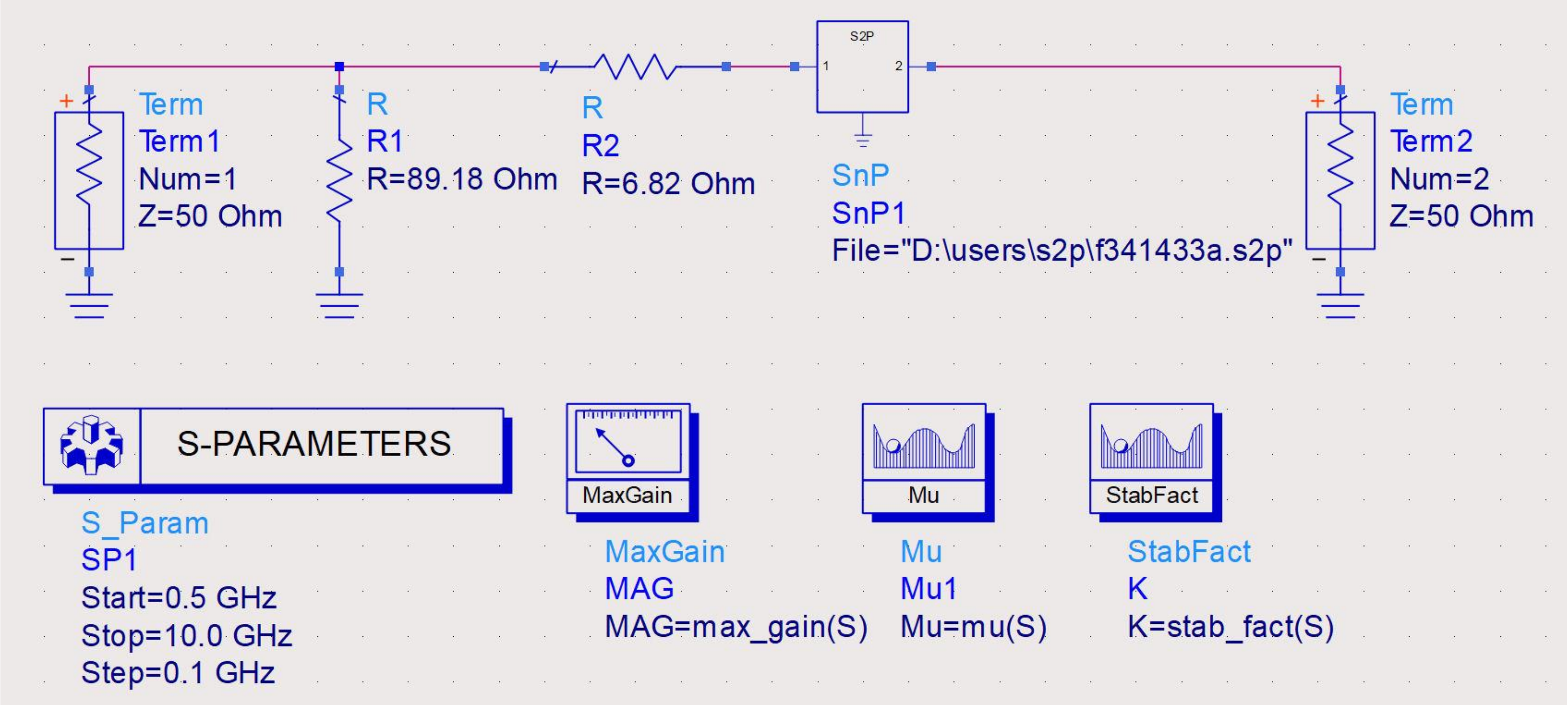


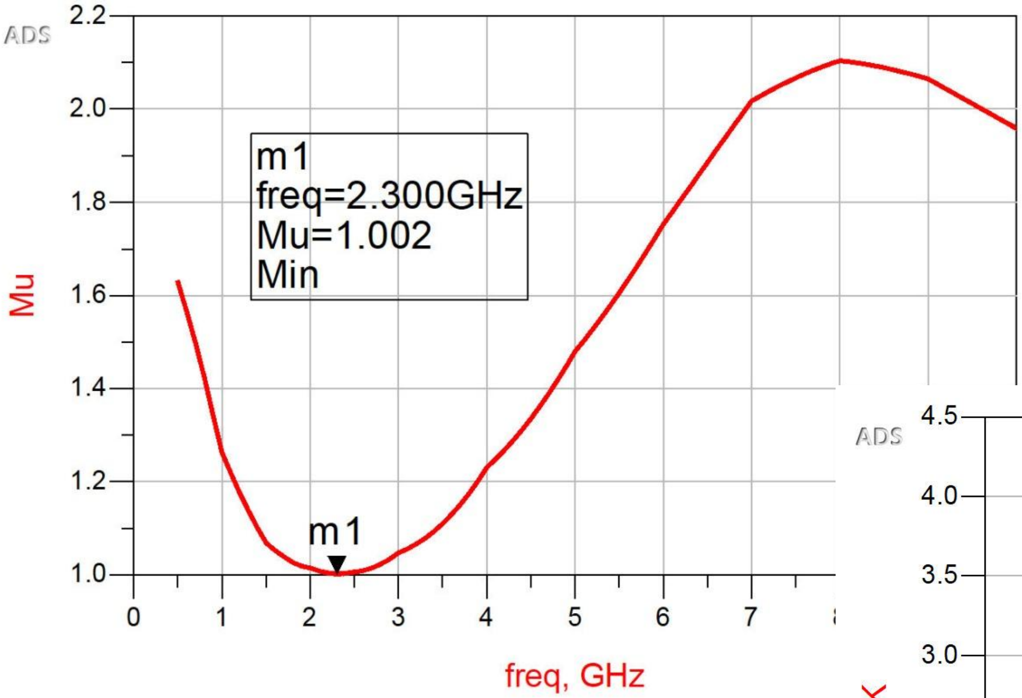
# Stabilization of two-port

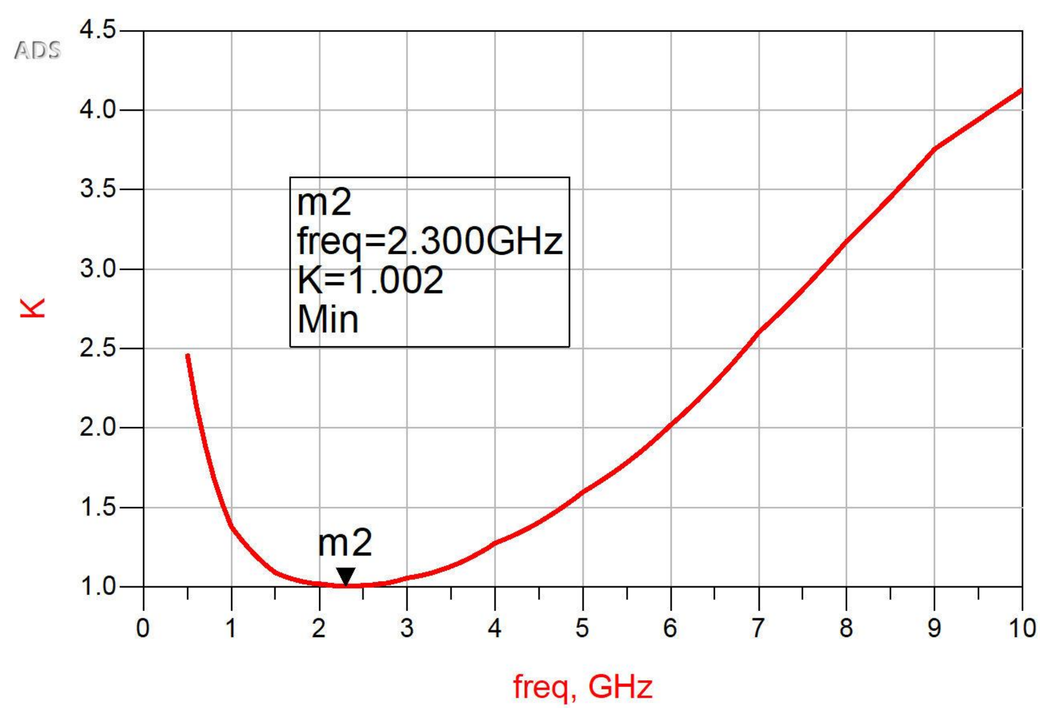
- Negative effect over the power gain
  - we must check MAG/MSG while designing resistive loading
- Negative effect over the noise (debated next)
- We can choose one of the 4 possibilities or a combination which offers better results (depending on transistor, application etc.)
- We can use frequency selective loading
  - Ex: RL, RC circuits which sacrifice performance only when needed to improve stability and have no effect at frequencies where the device is already stable
- It might be possible (and should be checked) that stability is improved as an effect of parasitic elements of biasing circuits (bypass capacitors and RF chokes)

# Stabilization of two-port

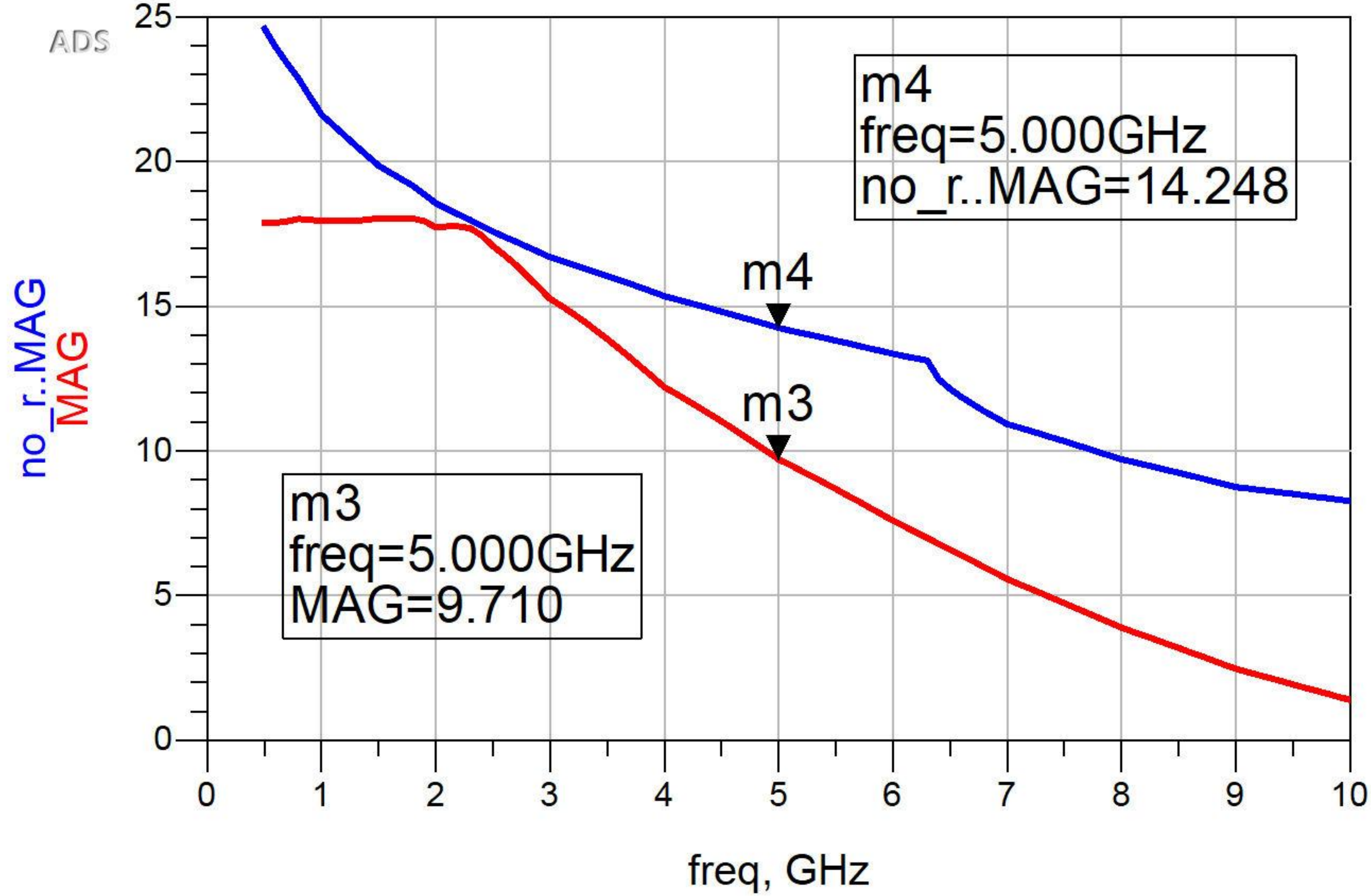


# Stabilization of two-port

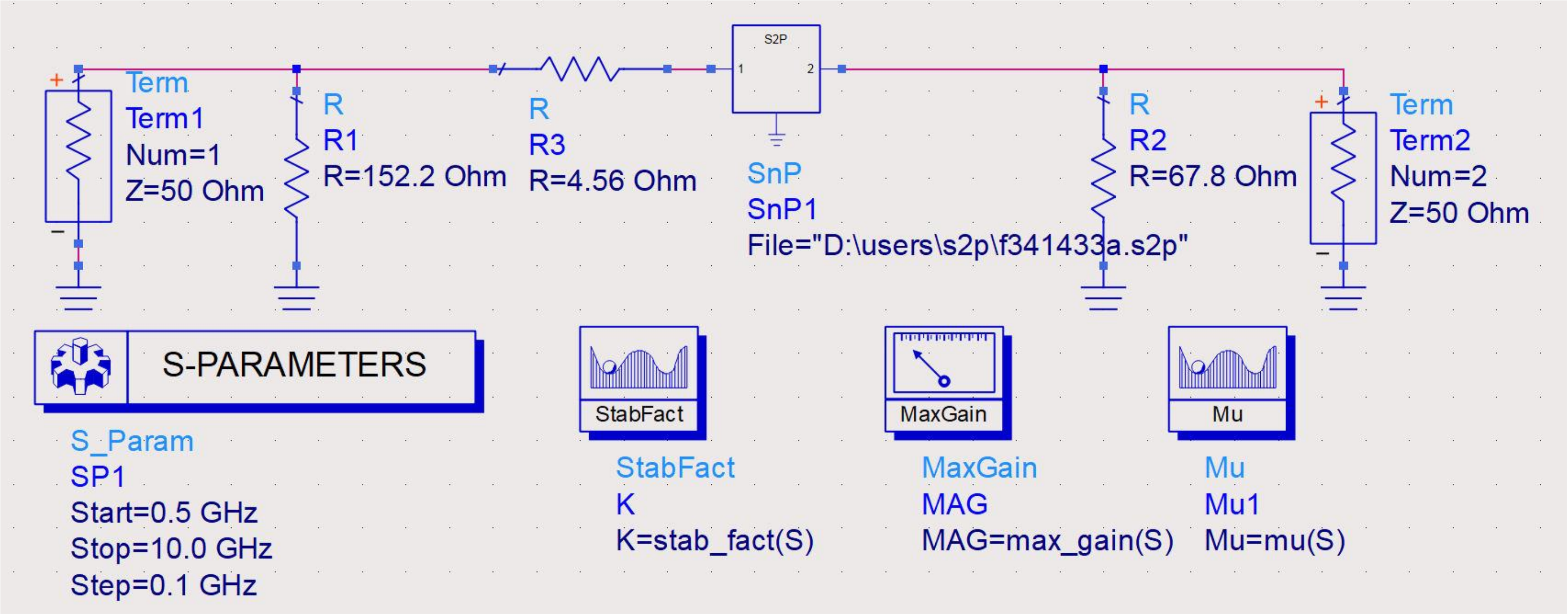


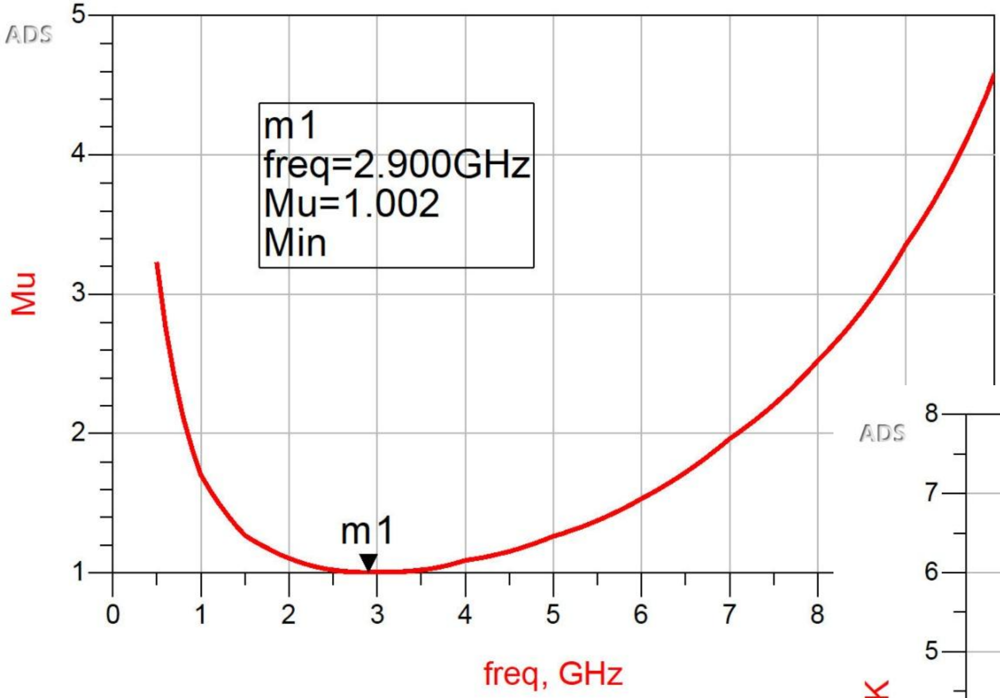
# Stabilization of two-port

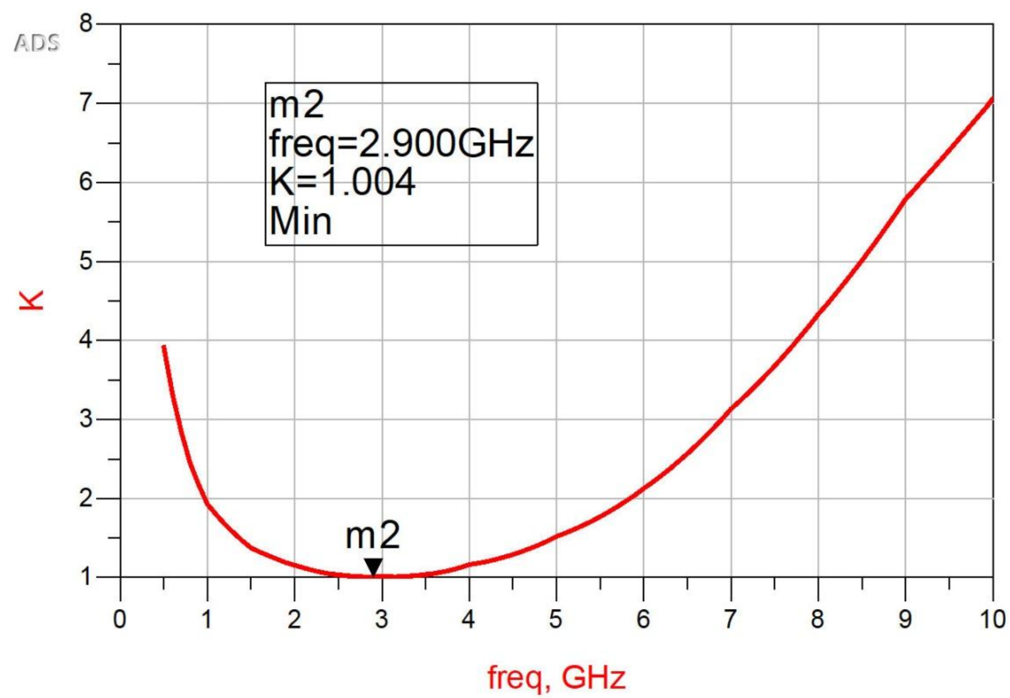


# Stabilization of two-port

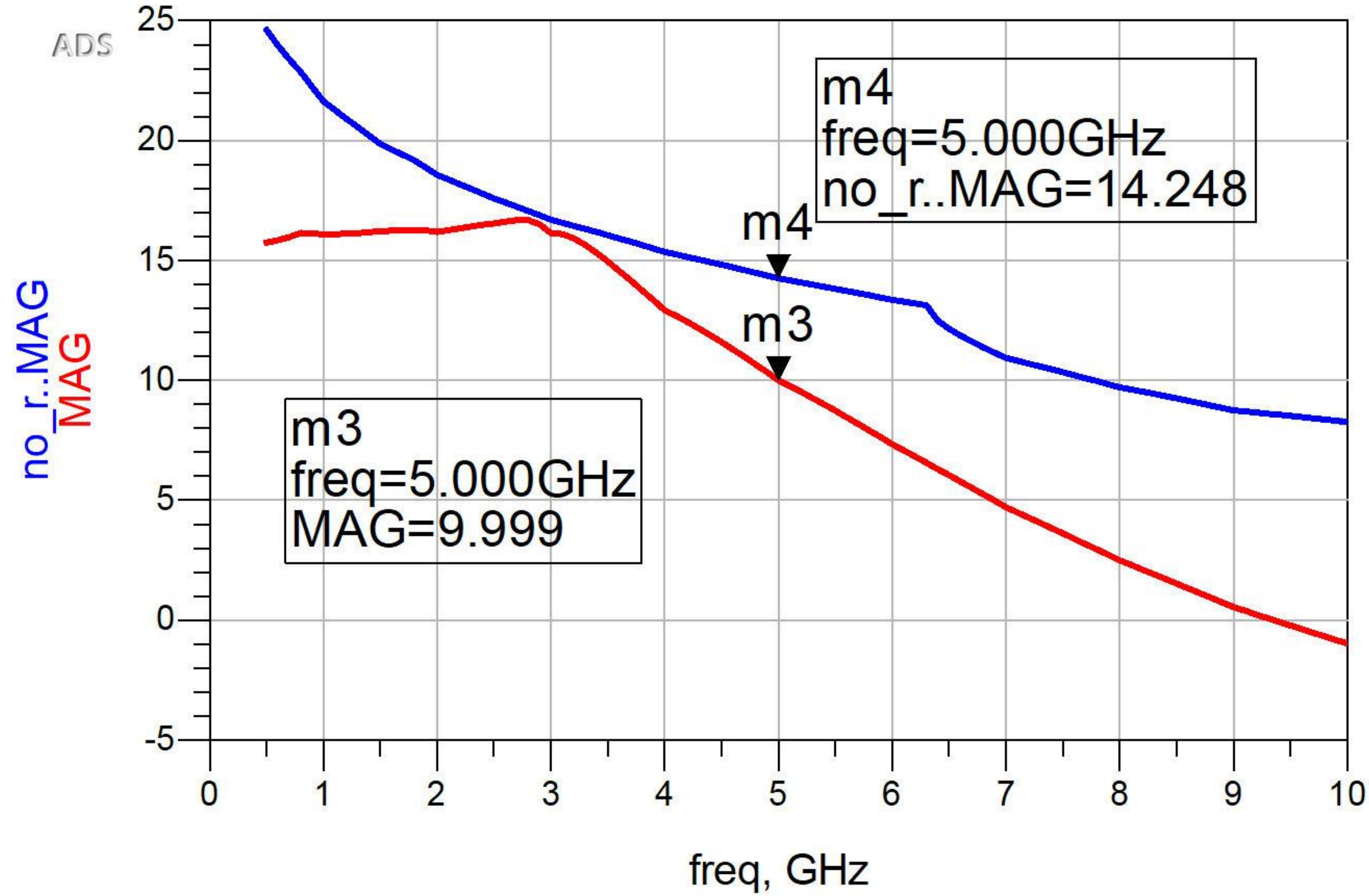


# Stabilization of two-port



# Stabilization of two-port



# Contact

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